SOLUTIONS

for Iverson's

ALGEBRA

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1

1.1 42; 27; 13; 13; 72; 72; 80; 1400; 33; 257; 257; 102; 94; 60; 95; 71

1.3 4; 4; 3; 3; 8; 7; 15; 4; 3; 5; 3

1.5 (7+1)×3; 17+(6×2); 5×(17×6); (3+2)+(8×5); (6+10)×(7+3); (4+14)+(3×13); 29+(19×6); (9+20)+(7+6); (8×3)+7; 15+(14+8); (6×6)+3; (1+2+3)×8; (3+4)×8; 2+(2×(9+5)); 6+(2×8)

1.6 Items 14 and 15 only:
14) The product of the quantities 2+3 and 4+6 added to the product of 2 and 5
15) 1 plus twice the sum of 3 and the quantity 4 times the sum of 5 and 6

1.7 14; 14; 47; 11; 27; 496; 8190; 40; 243; 44; 111; 155; 33

1.9 20; 20; 6; 6; 8; 48; 10; 48; 9; 2; 7

1.11 3×7+1; 17+6×2; 5×17×6; 3+2+8×5; (6+10)×7+3; 4+14+3×13; 29+19×6; 9+20+7+6; 7+8×3; 15+14+8; 3+6×6; 8×1+2+3; 8×3+4; 2+2×9+5; 6+2×8

1.12 Items 10 and 11 only:
10) 23 plus the quantity 7 times 2+1
11) 1 plus the sum of the quantities 9×11 and the product of 11 and 1

1.13 5; 6; 5; 76; 26; 29; 36; 8; 8; 8; 18; 300; 420; 200; 24; 24; 24; 24; 5; 25; 106; 39; 6; 1; 101; 49; 49; 49; 49; 175; 45; 30; 3; 9; 25

1.15 22; 4; 4; 1; 2; 12; 2; 6; 5; 12

1.17

```
L←100              BAR←20
W←50               WEIGHT←50
A←L×W              T←BAR+2×WEIGHT

A←3          N←5            X←90
B←4          D←10           Y←30
C←5          Q←N+2×D        NH←X+Y
P←A+B+C

D←500                  J←100
TIME←6                 S←1
TOTAL←D×TIME           WEIGHT←J+3×S
```

1.18 A dealer sold 100 bikes worth 50 dollars each. The money he received is the price of the bike times how many he sold.

A boy sawed a board into 3 pieces. One piece was 7 inches long, another was 2 feet long and the third was 4 yards long. The total length of board in inches was the number of inches plus 12 times the number of feet plus 36 times the number of yards.

1.19 54; 144; 700; 78; 47; 90; 118; 117; 34; 9; 6; 3; 9; 6; 3; 29; 82; 56; 7; 168; 9; 3840; 15; 1 3 5; 30; 12; 8; 4; 2; 8; 4; 2; 1805; 20; 112; 3; 109; 65; 17; 60; 57; 63; 99; 5; 92; 55404; 211680; 63; 9

1.20 +/4 6 8 9; ×/2 4 6; +/20 15 4; 6+×/4 1 2; 2++/3 12 4 20 or +/2 3 12 4 20; ×/3 7; 10××/8 3 or ×/10 8 3; 4++/3 7 or +/4 3 7; 3×+/1 2 3 4 5 6; ×/6 7 1 3; (+/4 3)×+/20 17 4 7; (+/3 4 5)××/2 8 3 4

1.21 Plus over 9 7 19 19; The product over 4 2 1 6 3; Times over 20 5 7; 18 plus the product over 20 3 1; The product of 2 and 4 increased by 39; The sum of 10 and 20 multiplied by 3; Plus over 43 7 19 21 28; The sum over 16 15 50 36; The sum of 30 and 4 ; 3 plus 3 plus 3

1.22 1 2 3 4; 10; 24; 1 2 3 4 5; 15; 120; 1; 1; 6; 10; 15; 21

1.23 4; 4; 5; 10; 4; 3; 8; 5; 1; 1; 10

1.24 ι3; ι5; ι9; +/ι3; ×/ι4; +/ι7; *Q*←4; ι*Q*; ι9

1.25 Items 1-3 only:
The first four integers; The sum over the first four integers; The product over the integers to 4

1.26 9 7 16 19; 5 5 5 5; 4 7 10 13; 34; 6 8 10 12; 6 8 10 12; 4 5 6 7; 5 10 15 20; 8 13 18 23; 1 4 9 16; 30; 7 9 11 13; 4 7 10 13; 9 25 49 81; 8 12 16 20; 63; 63; 15 18 21 24 27; 15 18 21 24 27

1.27 3 7 1 3; 4; 3 4 4 5; 5; 5; 20; 7 and 5; 7 and 5; 3 and 5 and 6; 5 and 6; 4 and 3 and 5; 12 and 4 and 5

1.28 4+ι5; 3×ι7; 4+3×ι9

1.29 5 5 5; 15; 15; 16; 1048576; 3 4 6 8; 3 4 6 8; 2 4 6 8; 1000000000; 1372; 24; 32 25 29 26

1.30 3; 8; 8; 6; 5; 10; 3; 8; 7; 10; 2; 7; 9; 5; 8; 8; 6; 10; 10; 9; 1; 3; 8; 1; 8; 4; 3; 6; 1

1.31 3ρ5; 5ρ3; +/6ρ4; ×/3ρ7; 7ρ6; +/10ρ4; (×/3 6)+2ρ5; 5 7 9×3ρ1; (4ρ7)+4ρ3; 3×6ρ5

1.32 10 10 11 12; 24 28 36 40; 96 77 60 45; 80 70 66 60; 18 27 45 63; 56 54 56 58; 78; 78; 102; 136 119 102 85; 52 78 130 182; 442; 13200; 1682 1683 1685 1687; 1890; 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17; 153; 21; 21; 362880; 3; 8 7 6 7; 8; 7; 12; 35; 15; 16 24 40 56; 28; 2352; 4 4 5 7; 20; 560

2

2.1 117; 130; 153; 120

2.2

(a)		(b)	(c)
57	114	*	¯1
58	116	*	¯1
59	118	*	¯1
60	120	*	0
61	122	*	1
62	124	*	2
63	126		4
64	128		6
65	130		7
66	132		9
67	134		11
68	136		13
69	138		15
70	140		17
71	142		19
72	144		21

2.3 123; 123; 122; 129 137 147; 141 158

2.4

(a)	
60	128
61	131
63	135
63	139
64	143
65	147
66	151

(b)			
66	133	141	151
67	137	145	155
68	141	149	158
69	145	153	162
70	149	157	165

(c)	
58	107
60	112
62	118
64	126
66	133
68	141

(d)			
67	137	145	155

2.5

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	a24
3	3	6	9	12	15	18	21	b24	27	30	33	36
4	4	8	12	16	20	c24	28	32	36	A40	44	48
5	5	10	15	20	25	30	35	B40	45	50	55	60
6	6	12	18	d24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	e24	32	C40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	D40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	f24	36	48	60	72	84	96	108	120	132	144

A 4×10 a 2×12 d 6×4
B 5×8 b 3×8 e 8×3
C 8×5 c 4×6 f 12×2
D 10×4

2.6

+	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	A9	10	11	12	13
2	3	4	5	6	7	8	B9	10	11	12	13	14
3	4	5	6	7	8	C9	10	11	12	13	14	15
4	5	6	7	8	D9	10	11	12	13	14	15	16
5	6	7	8	E9	10	11	12	13	14	15	16	17
6	7	8	F9	10	11	12	13	14	15	16	17	18
7	8	G9	10	11	12	13	14	15	16	17	18	19
8	H9	10	11	12	13	14	15	16	17	18	19	a20
9	10	11	12	13	14	15	16	17	18	19	b20	21
10	11	12	13	14	15	16	17	18	19	c20	21	22
11	12	13	14	15	16	17	18	19	d20	21	22	23
12	13	14	15	16	17	18	19	e20	21	22	23	24

A 1+8 E 5+4 a 8+12
B 2+7 F 6+3 b 9+11
C 3+6 G 7+2 c 10+10
D 4+5 H 8+1 d 11+9
e 12+8

2.7 2×*Y*; *X*×10; 5×*Y*; *X*×5

2.8
```
 2  8     a) 16; 24; 40
 4 16     b) 2×⍳8
 6 24     c) 8×⍳8
 8 32     d) 4×X (or X×4)
10 40
12 48
14 56
16 64
```

2.9 Note: Use Figure 2.3 as extended in Ex. 2.5.
```
 3  9     a) DOMAIN ERROR;
 6 18        18; DOMAIN ERROR
 9 27     b) 3×⍳10
12 36     c) 9×⍳10
15 45     d) 3×X (or X×3)
18 54
21 63
24 72
27 81
30 90
```

2.10

4	10
5	11
6	12
7	13
8	14
9	15
10	16
11	17
12	18
13	19
14	20
15	21

a) 10; 12; 16
b) 3+⍳12
c) 9+⍳12
d) 6+*X* (or *X*+6)

2.11 (1-3) Yes
4) 5; Yes
5) Yes, each column counts by its argument.
6) Yes, they agree for as far as they go and would agree entirely if the left and right domains were alike.
7) No, but they could be in any multiplication table in which the two domains are the same (See part (6) above); Yes
8) Only 5 unless we allow the non-positive integers introduced in the next chapter; Yes in a diagonal (/)-See exercise 2.6; Yes
9) If *R* is one row then the next row equals *R*+⍳*N* when *N* is the number of elements in the row.

2.12

a)

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20

b)

2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9

c)

1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16
5	10	15	20

d)

2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8
6	7	8	9

e)

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25

f)

2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

2.13

a)

1	2	3	4
2	4	6	8
3	6	9	12

b)

3	4	5
5	6	7
7	8	9
9	10	11
11	12	13

c)

4	6	8	10	12
6	8	10	12	14
8	10	12	14	16
10	12	14	16	18
12	14	16	18	20

d)

2	4	6	8
4	8	12	16
6	12	18	24

e)

6	7	8	9
7	9	11	13
8	11	14	17

f)

4	6	8	10	12
6	8	10	12	14
8	10	12	14	16
10	12	14	16	18
12	14	16	18	20

2.14

a)

H	1	2	3	4	5	6
1	4	5	6	7	8	9
2	7	8	9	10	11	12
3	10	11	12	13	14	15
4	13	14	15	16	17	18

b) 14; *DOMAIN ERROR*; 4; 16; 16; 11

2.15 (a,c) Same as Figure 2.2 but with right domain as 1 2 3 instead of 'small-frame medium-frame large frame'.
b) 141; 149; 139

2.16

a)

PLUS	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

b) 8; 10; 30; 8; 32; 14; 9; 9; 9; 9

2.17 3; 3; 8; 3; 14; 9; 10; 3; 2

2.18 14; 14; 6; 19; 2; 8; *LENGTH ERROR*; 54; 55; *LENGTH ERROR*; *LENGTH ERROR*; *LENGTH ERROR*;

A∘.⌈*B*

17	10	13	10	19
17	8	13	8	19
17	6	13	6	19
17	14	14	14	19
17	7	13	7	19
17	9	13	9	19

B∘.⌈*A*

17	17	17	17	17	17
10	8	6	14	7	9
13	13	13	14	13	13
10	8	6	14	7	9
19	19	19	19	19	19

B∘.⌊*A*

10	8	6	14	7	9
4	4	4	4	4	4
10	8	6	13	7	9
2	2	2	2	2	2
10	8	6	14	7	9

2.19 a) 12; 12; 15; 15; 100 ; 100

b) 18, that is (3×6); 400; 1300; 400; 1024000

2.20 8; 8; 32; 32; 4096; 4096; 1024; 1024; 100; 100

2.21 1 1 1 1 1 1 1 1; 4 8 16 32 64 128 256 512; 9 27 81 243 729 2187 6561 19783; 16 64 256 1024 4096 16384 65536 262144;

A∘.⋆*A*

4	8	16	32	64	128	256	512
9	27	81	243	729	2187	6561	19683
16	64	256	1024	4096	16384	65536	262144
25	125	625	3125	15625	78125	390625	1953125
36	216	1296	7776	46656	279936	1679616	10077696
49	343	2401	16807	117649	823543	5764801	40353607
64	512	4096	32768	262144	2097152	16777216	134217728
81	729	6561	59049	531441	4782969	43046721	387420489

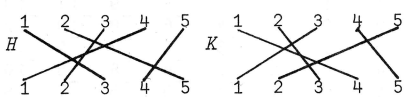
2.22 1 4 9 16 25 36; 1 8 27 64 125 216; 1 16 81 256 625 1296

2.23 a) 7; 9; 10; 10; 9; *DOMAIN ERROR*; 7 5 9 6 4 10 8

b) 8; 7; 4; 4; 7; 8 5 7 4 10 6 9; 4; 4; 6; 6; 4 5 6 7 8 9 10; 4 5 6 7 8 9 10

c) They are inverses- *F G X* is *X* and *G F X* is *X*. One function undoes the other.

d)



e-f)

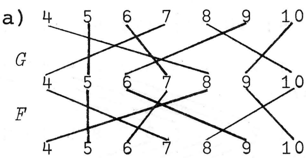
	F
4	7
5	5
6	9
7	6
8	4
9	10
10	8

	G
4	8
5	5
6	7
7	4
8	10
9	6
10	9

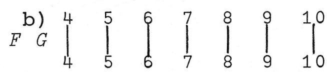
	H
1	3
2	5
3	2
4	1
5	4

	K
1	4
2	3
3	1
4	5
5	2

2.24 a)

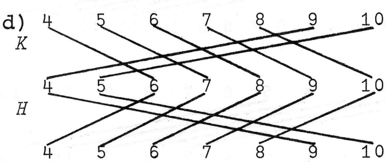


b)



c) The result is the same as the argument; such a function is called an <u>identity</u> function.

d)



parts b and c remain the same.

3

3.1 2; 7; 7 8 9 10 11 12; 1 2 3 4 5; 6 7 8 9 10; 7 6 5 4; 22; 2 5 5 10 23; 14 19 9 12 63; 8 12 7 11 43;

```
       M∘.+N
14 15 10  9 28
18 19 14 13 32
13 14  9  8 27
17 18 13 12 31
49 50 45 44 63
```

30; 30; 42; 42; 56; 56; 1 2 3 4 5; 5 4 3 2 1; 15; 15; 30; 6 6 6 6; 30;

```
       P∘.-ι5
 7  6  5  4  3
 8  7  6  5  4
 9  8  7  6  5
10  9  8  7  6
11 10  9  8  7
```

3.2 3; 4; 4; 8 12 5 13; impossible; 5; 10 10 19 9; 8

3.3 4; 10;17; 17; 4; 4; 4; 4

3.4

```
     1 2 3 4 5 6 7 8 9 10 11 12
a)
     1 2 3 4 5 6 7 8 9 10 11 12
b)
     1 2 3 4 5 6 7 8 9 10 11 12
```

```
 1 2 3 4 5 6 7 8 9 10 11 12 13 14
c)
 1 2 3 4 5 6 7 8 9 10 11 12 13 14
d)
 1 2 3 4 5 6 7 8 9 10 11 12 13 14
```

```
e)
1  2  3  4  5  6  7  8  9  10  11

1  2  3  4  5  6  7  8  9  10  11

1  2  3  4  5  6  7  8  9  10  11
```

3.5 ¯3; 4 3 2 1 0 ¯1 ¯2 ¯3; 0 ¯1 ¯2 ¯3 ¯4 ¯5 ¯6 ¯7; 7 6 5 4 3 2 1 0; ¯1 ¯2 ¯3 ¯4 ¯5 ¯6 ¯7 ¯8; 2 4 6 8 10 12 14 16; 0 0 0 0 0 0 0 0;

```
      S∘.-S
0 ¯1 ¯2 ¯3 ¯4 ¯5 ¯6 ¯7
1  0 ¯1 ¯2 ¯3 ¯4 ¯5 ¯6
2  1  0 ¯1 ¯2 ¯3 ¯4 ¯5
3  2  1  0 ¯1 ¯2 ¯3 ¯4
4  3  2  1  0 ¯1 ¯2 ¯3
5  4  3  2  1  0 ¯1 ¯2
6  5  4  3  2  1  0 ¯1
7  6  5  4  3  2  1  0
```

```
      T∘.-T
 0  ¯2  ¯4  ¯6  ¯8 ¯10 ¯12 ¯14
 2   0  ¯2  ¯4  ¯6  ¯8 ¯10 ¯12
 4   2   0  ¯2  ¯4  ¯6  ¯8 ¯10
 6   4   2   0  ¯2  ¯4  ¯6  ¯8
 8   6   4   2   0  ¯2  ¯4  ¯6
10   8   6   4   2   0  ¯2  ¯4
12  10   8   6   4   2   0  ¯2
14  12  10   8   6   4   2   0
```

```
      T∘.-S
 1   0  ¯1  ¯2  ¯3  ¯4  ¯5  ¯6
 3   2   1   0  ¯1  ¯2  ¯3  ¯4
 5   4   3   2   1   0  ¯1  ¯2
 7   6   5   4   3   2   1   0
 9   8   7   6   5   4   3   2
11  10   9   8   7   6   5   4
13  12  11  10   9   8   7   6
15  14  13  12  11  10   9   8
```

```
      S∘.-T
¯1  ¯3  ¯5  ¯7  ¯9 ¯11 ¯13 ¯15
 0  ¯2  ¯4  ¯6  ¯8 ¯10 ¯12 ¯14
 1  ¯1  ¯3  ¯5  ¯7  ¯9 ¯11 ¯13
 2   0  ¯2  ¯4  ¯6  ¯8 ¯10 ¯12
 3   1  ¯1  ¯3  ¯5  ¯7  ¯9 ¯11
 4   2   0  ¯2  ¯4  ¯6  ¯8 ¯10
 5   3   1  ¯1  ¯3  ¯5  ¯7  ¯9
 6   4   2   0  ¯2  ¯4  ¯6  ¯8
```

4; ¯2; 3; ¯3; 4; ¯4

3.6 11; 5; 48; 3 2 10 20; 5; 4 ¯1 6 1 8; 5; 9; 13; 6; 8; 16

3.7 (a-b)

¯2 ¯1 0 1 2 3 4 5

¯2 ¯1 0 1 2 3 4 5

(c-d)

¯8 ¯7 ¯6 ¯5 ¯4 ¯3 ¯2 ¯1 0 1 2 3 4 5 6 7

¯8 ¯7 ¯6 ¯5 ¯4 ¯3 ¯2 ¯1 0 1 2 3 4 5 6 7

(e-f)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

3.8 ¯2 ¯1 0 1 2; ¯2 ¯1 0 1 2; 10 11 12 13 14 15 16; 10 11 12 13 14 15 16; ¯1 ¯2 ¯3 ¯4 ¯5 ¯6; 1 2 3 4 5 6; 0 0 0 0 0 0; 2 4 6 8 10 12; ¯2 ¯4 ¯6 ¯8 ¯10 ¯12;

P∘.+*N*

0	¯1	¯2	¯3	¯4	¯5
1	0	¯1	¯2	¯3	¯4
2	1	0	¯1	¯2	¯3
3	2	1	0	¯1	¯2
4	3	2	1	0	¯1
5	4	3	2	1	0

P∘.-*N*

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

N∘.+*P*

0	1	2	3	4	5
¯1	0	1	2	3	4
¯2	¯1	0	1	2	3
¯3	¯2	¯1	0	1	2
¯4	¯3	¯2	¯1	0	1
¯5	¯4	¯3	¯2	¯1	0

N∘.-*P*

¯2	¯3	¯4	¯5	¯6	¯7
¯3	¯4	¯5	¯6	¯7	¯8
¯4	¯5	¯6	¯7	¯8	¯9
¯5	¯6	¯7	¯8	¯9	¯10
¯6	¯7	¯8	¯9	¯10	¯11
¯7	¯8	¯9	¯10	¯11	¯12

3.9 5 ¯4 5 ¯13; ¯4 ¯6 0 ¯5 7; ¯4 ¯6 0 ¯5 7; 22 7 ¯5 1 ¯5; 3; ¯3 ¯3 ¯3 ¯1 ¯3; ¯8 ¯5 ¯3 2 12; 8 5 3 ¯2 ¯12; 4; 6; 5; 6; 7; 8

3.10 ¯9+⍳17; ¯5+⍳20; 3×¯5+⍳9; 1+2×¯6+⍳10; ⍳6; 7-⍳6; ¯7+⍳6; 0-⍳6

4

4.1 a)

-	1	2	3	4	5	6	7	8	9	10	11	12
1	0	¯1	¯2	¯3	¯4	¯5	¯6	¯7	¯8	¯9	¯10	¯11
2	1	0	¯1	¯2	¯3	¯4	¯5	¯6	¯7	¯8	¯9	¯10
3	2	1	0	¯1	¯2	¯3	¯4	¯5	¯6	¯7	¯8	¯9
4	3	2	1	0	¯1	¯2	¯3	¯4	¯5	¯6	¯7	¯8
5	4	3	2	1	0	¯1	¯2	¯3	¯4	¯5	¯6	¯7
6	5	4	3	2	1	0	¯1	¯2	¯3	¯4	¯5	¯6
7	6	5	4	3	2	1	0	¯1	¯2	¯3	¯4	¯5
8	7	6	5	4	3	2	1	0	¯1	¯2	¯3	¯4
9	8	7	6	5	4	3	2	1	0	¯1	¯2	¯3
10	9	8	7	6	5	4	3	2	1	0	¯1	¯2
11	10	9	8	7	6	5	4	3	2	1	0	¯1
12	11	10	9	8	7	6	5	4	3	2	1	0

b) Each diagonal contains the same number repeated. The main diagonal is zero, the lower diagonals are negative, and the upper are positive.

4.2 a)

```
 ⍉S           ⌽S
 0  1  2      ¯3 ¯2 ¯1  0
¯1  0  1      ¯2 ¯1  0  1
¯2 ¯1  0      ¯1  0  1  2
¯3 ¯2 ¯1

 ⊖S              B∘.-A        0-B∘.-A
2  1  0 ¯1       0 ¯1 ¯2       0  1  2
1  0 ¯1 ¯2       1  0 ¯1      ¯1  0  1
0 ¯1 ¯2 ¯3       2  1  0      ¯2 ¯1  0
                 3  2  1      ¯3 ¯2 ¯1
```

b) 0-B∘.-A

c)

```
 ⌽B                 ⌽A
4 3 2 1             3 2 1

 ⌽S                 (⌽A)∘.-B
¯3 ¯2 ¯1  0         2  1  0 ¯1
¯2 ¯1  0  1         1  0 ¯1 ¯2
¯1  0  1  2         0 ¯1 ¯2 ¯3

 A∘.-⌽B              ⊖S
¯3 ¯2 ¯1  0         2  1  0 ¯1
¯2 ¯1  0  1         1  0 ¯1 ¯2
¯1  0  1  2         0 ¯1 ¯2 ¯3
```

d) The flip (⌽) of the right domain yields ⌽ of the original table; ⌽ of the left domain yields ⊖ of the original table.

e) ⊖⌽S

4.3 a)

```
⌽⊖M     ⊖⌽M     ⍉⌽M
4 3     4 3     2 4
2 1     2 1     1 3

⌽⍉M     ⍉⊖M     ⊖⍉M     ⌽⍉⌽M
3 1     3 1     2 4     4 2
4 2     4 2     1 3     3 1
```

b) 8 (including the identity which yields the original, as in ⌽⌽M)

c) No, and there are 15 others which cannot be produced by flipping.

d) Take a piece of paper and write "front" on one side and "back" on the other. Then write the numbers in the appropriate corners on each side. Then see how many positions you can put the piece of paper into. There are only 8 positions.

e) M; ⌽⍉M; ⊖⌽M; ⍉⌽M; ⌽M; ⌽⍉⌽M; ⊖M; ⍉M

f) Because S is a table in which all elements in any downward sloping (to the right) diagonal are alike; S can be flipped about the upward slope diagonal (by the expression ⌽⍉⌽S) without changing it. But a table flipped in this way and then flipped by ⌽⊖ is equivalent to the flip ⍉S.

4.4 a) 7; 4; ¯4; 2; 2; ¯4; 3 1
¯1 ¯3 ¯5; ¯4 ¯3 ¯2 ¯1 0 1; ¯4

b) 5 7; 4 5 6; 7 8 9

```
M[2 4;1 3 5]
3 ¯1 ¯5
5  1 ¯3

M[2 4;]
3 1 ¯1 ¯3 ¯5
5 3  1 ¯1 ¯3

M[;1 3]
2 ¯2
3 ¯1
4  0
5  1
6  2
7  3
```

6; 8; 14; 14

4.5 1) Each number in the table shows only in one particular diagonal, that is 0=B or ¯3=B, etc. yield tables with one upward diagonal of 1's.
2) The remarks in solution to Exercise 4.3 f) apply to upward diagonals in table B
3) All negative numbers occur above the diagonal, that is 0>B yields an upper triangular table of 1's.
4) A flip about the upward sloping diagonal (⌽⍉⌽B) yields the negative of B, that is, B+⌽⍉⌽B yields a table of zeros.

4.6 1) ⍉N and ⊖⌽N are each equal to N
2) N+⊖N and N+⌽N each yield a table of zeros
3) Any row of N is a "counting by 2" or "counting by 3", etc.; that is, N[I;] is counting by J[I].

4.7 a) Q1←N[⍳7;8+⍳7]
Q3←N[8+⍳7;⍳7]

b) Q1 is equal to ⍉Q3
Q2 is equal to ⌽⍉⌽Q4
Q1 is the negative of ⌽Q2 and of ⊖Q4
Q3 is the negative of ⊖Q2 and of ⌽Q4

4.8 1) MAX is equal to ⍉MAX
2) Any number in the table shows in an L-shaped pattern, for example ¯4=MAX yields a reversed L-shaped pattern of 1's.

4.9 1) See Exercise 4.8
2) ⌽⍉⌽MAX equals -MIN

4.10

```
I∘.⌈I
1 2 3 4 5 6
2 2 3 4 5 6
3 3 3 4 5 6
4 4 4 4 5 6
5 5 5 5 5 6
6 6 6 6 6 6

J∘.⌊J
¯1 ¯2 ¯3 ¯4 ¯5 ¯6
¯2 ¯2 ¯3 ¯4 ¯5 ¯6
¯3 ¯3 ¯3 ¯4 ¯5 ¯6
¯4 ¯4 ¯4 ¯4 ¯5 ¯6
¯5 ¯5 ¯5 ¯5 ¯5 ¯6
¯6 ¯6 ¯6 ¯6 ¯6 ¯6

I∘.⌊I
1 1 1 1 1 1
1 2 2 2 2 2
1 2 3 3 3 3
1 2 3 4 4 4
1 2 3 4 5 5
1 2 3 4 5 6

J∘.⌈J
¯1 ¯1 ¯1 ¯1 ¯1 ¯1
¯1 ¯2 ¯2 ¯2 ¯2 ¯2
¯1 ¯2 ¯3 ¯3 ¯3 ¯3
¯1 ¯2 ¯3 ¯4 ¯4 ¯4
¯1 ¯2 ¯3 ¯4 ¯5 ¯5
¯1 ¯2 ¯3 ¯4 ¯5 ¯6
```

4.11 b) The numbers in table T form concentric boxes

4.12 0; 1; 0 0 0 1 0 0 0; 1 1 1 0 1 1 1;

```
X∘.=X
1 0 0 0 0 0 0
0 1 0 0 0 0 0
0 0 1 0 0 0 0
0 0 0 1 0 0 0
0 0 0 0 1 0 0
0 0 0 0 0 1 0
0 0 0 0 0 0 1

X∘.≠X
0 1 1 1 1 1 1
1 0 1 1 1 1 1
1 1 0 1 1 1 1
1 1 1 0 1 1 1
1 1 1 1 0 1 1
1 1 1 1 1 0 1
1 1 1 1 1 1 0

X∘.=Y
0 0 0 0 0 0 1
0 0 0 0 0 1 0
0 0 0 0 1 0 0
0 0 0 1 0 0 0
0 0 1 0 0 0 0
0 1 0 0 0 0 0
1 0 0 0 0 0 0
```

4.13

```
X∘.>X
0 0 0 0 0 0 0
1 0 0 0 0 0 0
1 1 0 0 0 0 0
1 1 1 0 0 0 0
1 1 1 1 0 0 0
1 1 1 1 1 0 0
1 1 1 1 1 1 0

X∘.≥X
1 0 0 0 0 0 0
1 1 0 0 0 0 0
1 1 1 0 0 0 0
1 1 1 1 0 0 0
1 1 1 1 1 0 0
1 1 1 1 1 1 0
1 1 1 1 1 1 1
```

```
      X∘.≤X                    X∘.≤⌽X
1 1 1 1 1 1 1            1 1 1 1 1 1 1
0 1 1 1 1 1 1            1 1 1 1 1 1 0
0 0 1 1 1 1 1            1 1 1 1 1 0 0
0 0 0 1 1 1 1            1 1 1 1 0 0 0
0 0 0 0 1 1 1            1 1 1 0 0 0 0
0 0 0 0 0 1 1            1 1 0 0 0 0 0
0 0 0 0 0 0 1            1 0 0 0 0 0 0

      ⍉X∘.≥X
1 1 1 1 1 1 1
0 1 1 1 1 1 1
0 0 1 1 1 1 1
0 0 0 1 1 1 1
0 0 0 0 1 1 1
0 0 0 0 0 1 1
0 0 0 0 0 0 1
```

4.14

```
      4≤A
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 1 1
0 0 0 0 0 0 0 0 1 1 1
0 0 0 0 0 0 0 1 1 1 1
0 0 0 0 0 0 1 1 1 1 1
0 0 0 0 0 1 1 1 1 1 1
0 0 0 0 1 1 1 1 1 1 1

      16≤A*2
1 1 1 1 1 1 1 0 0 0 0
1 1 1 1 1 1 0 0 0 0 0
1 1 1 1 1 0 0 0 0 0 0
1 1 1 1 0 0 0 0 0 0 0
1 1 1 0 0 0 0 0 0 0 1
1 1 0 0 0 0 0 0 0 1 1
1 0 0 0 0 0 0 0 1 1 1
0 0 0 0 0 0 0 1 1 1 1
0 0 0 0 0 0 1 1 1 1 1
0 0 0 0 0 1 1 1 1 1 1
0 0 0 0 1 1 1 1 1 1 1

      4≤S
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0
1 1 0 0 0 0 0 0 0 0 0
1 1 1 0 0 0 0 0 0 0 0
1 1 1 1 0 0 0 0 0 0 0
1 1 1 1 1 0 0 0 0 0 0
1 1 1 1 1 1 0 0 0 0 0
1 1 1 1 1 1 1 0 0 0 0

      12≥M
0 0 0 1 1 1 1 1 1 1 1
0 0 1 1 1 1 1 1 1 1 1
0 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 0
1 1 1 1 1 1 1 1 1 0 0
1 1 1 1 1 1 1 1 0 0 0

      144≥M*2
0 0 0 1 1 1 1 1 0 0 0
0 0 1 1 1 1 1 1 1 0 0
0 1 1 1 1 1 1 1 1 1 0
1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1
0 1 1 1 1 1 1 1 1 1 0
0 0 1 1 1 1 1 1 1 0 0
0 0 0 1 1 1 1 1 0 0 0
```

4.15

```
0 0 1 1 0 0; 1; 0; 1 1 1 1
1 1; 1; 1; 1; 0; 0; 0
```

4.16

```
      A=⍉A             S=⍉S
1 1 1 1 1 1       1 0 0 0 0 0
1 1 1 1 1 1       0 1 0 0 0 0
1 1 1 1 1 1       0 0 1 0 0 0
1 1 1 1 1 1       0 0 0 1 0 0
1 1 1 1 1 1       0 0 0 0 1 0
1 1 1 1 1 1       0 0 0 0 0 1

1 1 1 1 1 1; 1; 0; 1 1 1 1 1 1;
1; 1 2 3 4 5 6; 6 5 4 3 2 1
```

5

5.1 32; 8; 4; 6; 10; 10; 6 12 18 24 30 36 42; 3 6 9 12 15 18 21; 2 4 6 8 10 12 14; 1 2 3 4 5 6 7; 6 12 18 24 30 36 42;

S∘.÷1 2 3 6

6	3	2	1
12	6	4	2
18	9	6	3
24	12	8	4
30	15	10	5
36	18	12	6
42	21	14	7

¯3 ¯6 ¯9 ¯12 ¯15 ¯18 ¯21; ¯6 ¯12 ¯18 ¯24; ¯18 ¯12 ¯6 0 6 12 18;

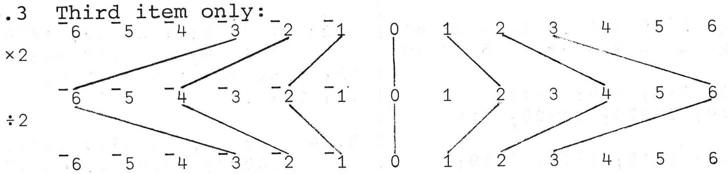
T∘.÷1 2 3 6

¯18	¯9	¯6	¯3
¯12	¯6	¯4	¯2
¯6	¯3	¯2	¯1
0	0	0	0
6	3	2	1
12	6	4	2
18	9	6	3

¯4 4 ¯2 3; 8 5 7 7; 4 9 5 10; 4 9 5 10; ¯12 ¯1 ¯9 ¯4

5.2 8; 8; 20; 20; 500; 500; 224; 224; 7; 7; 172; ¯200; ¯228; 228; ¯172; 11 3 2; 1

5.3 Third item only:

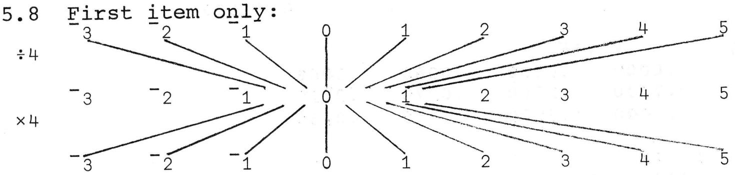


5.4 32; 54; 256; 120; 384; 2520; 13440; 7560; 10080

5.5 327; 658; 3779; 2294; 2664; 1095; 236

5.7 1358; 2718; 3739; 216; 2646; 208581831; 5147; 268; 268; 23875; 2387; 144; 12; 1

5.8 First item only:



5.9

DIVISOR	4	2	14	64	144
DIVIDEND	8	10	196	2048	1728
QUOTIENT	2	5	14	32	12

5.10

NUMERATOR	8	10	196	2048	1728
DENOMINATOR	4	2	14	64	144

5.11 one half; one third; two fifths; seven fifths; two sixths; three sixths; four sixths; six sixths; negative-seven twelfths

5.12 and 5.13 The three rows represent the Numerator, Denominator, and Integer in that order:

14	26	26	0	48	¯9	39	36	¯19	¯2	37	0	24	6	6
7	13	13	15	3	19	7	9	15	9	1	12	18	11	11
2	2	2	0	16	--	--	4	--	--	37	0	--	--	--

5.14 14; 21; ¯8; ¯55; 9; 0; 12

5.15 10÷21; 15÷15; ¯510÷34; 4÷9; 88÷63; 403÷48; 178÷24; 208÷60; 10÷24; 0÷12; ¯35÷25; 36÷48

5.16 10÷21; 1; ¯15; 4÷9; 88÷63; 403÷48; 178÷24; 208÷60; 10÷24; 0; ¯35÷25; 36÷48

5.17 NOTE: an * following any entry indicates that many pairs of values are possible.

1st blank	6	15	4	5	6*	12	3	23	41
2nd blank	-	-	8	4	9*	5	5	17	39

5.18 2÷3; 2÷3; 3÷4; 13÷9; 13÷9; 13÷9; 27÷20; 27÷20; 7÷6

5.19 12÷5; 12÷5; 15÷5; 77÷9; 63÷9; 21÷27; 7÷9; 70÷26; 2÷12; 8÷3; 12÷2

5.20 ÷/160 560; ÷/160 560; ÷/¯120 21; ÷/92 92; ÷/48 100; ÷/8 12; ÷/8 12; ÷/14 28; ÷/49 29; ÷/6 35; ÷/6 35; ÷/8 5; ÷/8 5; ÷/40 21

5.21 Note: parts 10 and 11 were stated incorrectly and solutions are not given for them. 38÷35; 38÷30; 132÷408; 132÷408; 276÷408; 35÷50; ÷/35 50; ÷/35 6; ÷/70 6; --; --; 25÷12; 23÷12; ÷/23 12

5.22 ÷/34 35; ÷/6 35; ÷/¯6 35; ÷/34 35; ÷/56 49; ÷/¯32 45; ÷/324 567; ÷/32 45; ÷/490 441; ÷/1246 441; ÷/656 315; ÷/8 35; ÷/1060 567; ÷/11236 3969; ÷/20 45; ÷/656 315

5.23 ÷/27 8; ÷/63 12; ÷/¯63 12; ÷/27 4; 6; ÷/18 20; ÷/20 18; ÷/60 72; ÷/234 1035; ÷/4 21; ÷/4 21; ÷/7 16

5.24 .5; .2; 8; .34; .034; 3.4; .0007; 23.4; 2.34; .234; 4.5; .0294; 3.8; 5.0 or 5; .23; ¯.008; ¯567; 100.00 or 100; 45.67; 28.345; .079; ¯.078; 29384.7; 29; 92.87654; .00009; .23; 3688.7

5.25 .098; .098; 1.776; .41; 2.00 or 2; 15.14; 28.42; 4.015224

5.26 *V*∘.÷*E*

.6000	.0600	.0060	.0006
2.7000	.2700	.0270	.0027
13.5000	1.3500	.1350	.0135

V∘.÷*F*

600.0000	60.0000	6.0000	.6000	.0600	.0060	.0006
2700.0000	270.0000	27.0000	2.7000	.2700	.0270	.0027
13500.0000	1350.0000	135.0000	13.5000	1.3500	.1350	.0135

5.27 40.6; 8.1; 16.2; 48.9; 9.4; 9.87; 112.1829; 609.61; 3.38; ¯64.2; 866.04; ¯6.4

5.28 13.57; 15.00; 792.1458; 69.5; 48.93; .96; 5.22; ¯260.19; 34.85; ¯760.3678; 2.357; 3.18; 614.9; 11.117; ¯73.779

5.29 .75; 69.12; 108; 12.75; .4 .12; .6 .12; .125 .25 .375 .5 .625 .75 .875 1; .0625 .125 .1875 .25 .3125 .375 etc; .03125 .0625 .09375 .125 .15625 etc; .04 .08 .12 .16 .2 .24 .28 .32 etc; .25 .5 .75 1 1.25 1.5 1.75 2 2.25 etc; .5 .25 .125 .0625 .03125 .015625; .2 .04 .008 .0016 .00032 .000064; .1 .01 .001 .0001 .00001 .000001; .875 .75 .625 .5 .375 .25 .125 0; .96875 .9375 .90625 .875 .84375 etc.

5.30 .333; .667; .111 .222 .333 .444 .556 .667 .778 .889 1; .031 .063 .094 .125 .156 .188 .219 .250 etc.;

```
      (ι10)∘.÷(ι10)
 1.000  .500  .333  .250  .200  .167  .143  .125  .111  .100
 2.000 1.000  .667  .500  .400  .333  .286  .250  .222  .200
 3.000 1.500 1.000  .750  .600  .500  .429  .375  .333  .300
 4.000 2.000 1.333 1.000  .800  .667  .571  .500  .444  .400
 5.000 2.500 1.667 1.250 1.000  .833  .714  .625  .556  .500
 6.000 3.000 2.000 1.500 1.200 1.000  .857  .750  .667  .600
 7.000 3.500 2.333 1.750 1.400 1.167 1.000  .875  .778  .700
 8.000 4.000 2.667 2.000 1.600 1.333 1.143 1.000  .889  .800
 9.000 4.500 3.000 2.250 1.800 1.500 1.286 1.125 1.000  .900
10.000 5.000 3.333 2.500 2.000 1.667 1.429 1.250 1.111 1.000
```

9.931; 27.048; 8.828; 1.270; .054; .903; 1.657; 1.298; .277; .097; 49.5; .563; 2.953

5.31 3.5668; 53.628; 18.431127; 2894.4704; 220.248; 138.1848; .36

5.32 3.57; 53.63; 18.43; 2894.47; 220.25; 138.18; .36

5.33 1.628; .199; .090; 19.008; .553; 1.278; 3.516

5.34 NOTE: Each part could be written in several different ways as well as those given below.
3.5668*E*0; 5.3628*E*1; 1.8431127*E*1; 2.8944704*E*3; 2.20248*E*2; 1.381848*E*2; 3.6*E*¯1

5.35 .001628*E*3; .000199*E*3; .000090*E*3; .019008*E*3; .000553*E*3; .001278*E*3; .003516*E*3

5.36 .667; ¯.667; ¯.667; .667

6

6.1 2 3 5 7 4 1 2; 4 1 2 2 3 5 7; 2 3 5 7 4 1 2 9 8; 2 3 5 7 4 1 2 9 8; ¯4 ¯3 ¯2 ¯1 1 2 3 4

6.2 a)

```
     D=1
1 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0
0 0 1 0 0 0 0 0
0 0 0 1 0 0 0 0
0 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 1
```

```
      D=1÷2                    D=1÷3
0 1 0 0 0 0 0 0        0 0 1 0 0 0 0 0
0 0 0 1 0 0 0 0        0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0        0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1        0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0        0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0        0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0        0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0        0 0 0 0 0 0 0 0
```

b) ones lie along a downward sloping diagonal

6.3 The four quadrants are given by [*A*+ι4;*B*+ι4], where *A* is 0 or 5 and *B* is 0 or 4

b) The following sets of expressions are equal:

```
  Q1 and ⌽⊖Q3 and ⌽-Q2 and ⊖-Q4
  Q2 and ⌽⊖Q4 and ⌽-Q1 and ⊖-Q3
  Q3 and ⌽⊖Q1 and ⌽-Q4 and ⊖-Q2
  Q4 and ⌽⊖Q2 and ⌽-Q3 and ⊖-Q1
```

6.4 a) .857 .875 .889 .9 .909 .917; .9 .909 .917 .923 .929 .933;

```
          T
1 1 1 1 1 1
1 1 1 1 1 1
1 1 1 1 1 1
1 1 1 1 1 1
0 1 1 1 1 1
0 0 1 1 1 1
```

b) The last of each pair

c) (*A*∘.×*D*)≤(*B*∘.×*C*)

6.5

```
      2 3∘.*1+ι10
4   8  16   32   64   128   256    512   1024    2048
9  27  81  243  729  2187  6561  19683  59049  177147
```

.125 .25 .5 1 2 4 8 16 32 64 128 256; .037 .111 .333 1 3 9 27 81 243 729 2187 6561;

```
      2 3∘.*¯4+ι12
.125 .250 .500  1  2  4   8  16   32   64   128   256
.037 .111 .333  1  3  9  27  81  243  729  2187  6561

      2 3 4 5 6∘.*¯4+ι7
.125    .250    .500   1.000   2.000   4.000    8.000
.037    .111    .333   1.000   3.000   9.000   27.000
.016    .063    .250   1.000   4.000  16.000   64.000
.008    .040    .200   1.000   5.000  25.000  125.000
.005    .028    .167   1.000   6.000  36.000  216.000
```

6.6 a) 2 4 8 16 32; .5 .25 .125 .0625 .03125; 1 1 1 1 1; 1 1 1 1 1 1 1; 100

b) 3 9 27 81; .33333 .11111 .03704 .01235; 1 1 1 1

c) 5 25 125 625; .2 .04 .008 .0016; .2 .04 .008 .0016; 1 1 1 1

6.7 10 100 1000 10000 100000; .1 .01 .001 .0001 .00001; ¯10 100 ¯1000 10000 ¯100000; ¯.1 .01 ¯.001 .0001 ¯.00001; 20 400 8000 160000 3200000; ¯20 400 ¯8000 160000 ¯3200000; 0 0 0 0 0; undefined (*DOMAIN ERROR*)

6.8 3 9 27 81 243 729; .333 .111 .037 .012 .004 .001; 7 49 343 2401 16807 117649; .143 .020 .003 .000 .000 .000

6.9 3.1623; 3.1623 10.0001 31.6234 100.0028 316.2389 1000.0424; .3163 .1000 .0316 .0100 .0032 .0010

6.10 1.442 2.08 3 4.327 6.24 9; 1.316 1.732 2.28 3 3.948 5.196; 1.246 1.552 1.933 2.408 3 3.737; 1.201 1.442 1.732 2.08 2.498 3; 1.308 1.71 2.236 2.924 3.824 5

7

```
7.1  2; 2 2 0 0; 1 2 3 4 5 6 7 8
     _0; 0; 3; 6; 66; 6 1_9 12 2
14; ¯658 ¯660 ¯656 ¯660 ¯656
¯658; 2; 0 1 0 0 1 0; 1; 0 2 5 5;
0 3 3 2 1; 0 5; 202 12 871; 14 4
11 12 14; 7; 4 3; 1510; 1; 0 5 0;
3; 2
```

7.2 NOTE: Answers listed in order across the page:

```
0 0 0 0      0 0 0 0 0      0 0 0 0
0 1 0 1      2 0 2 0 0      2 0 0 3
1 2 0 1      0 0 0 0 0


0 0 2       1 1 1 1        2  6  0
3 2 0       1 0 1 1        2  6  0
0 4 2       1 1 1 1        0  6  8
0 0 2       1 1 1 1       65  6  8
            1 1 1 1        2  0  2

0 8 4 0 7   4  8   0 0   3 0 0 6
1 2 2 1 1   4 52   3 1   3 1 4 5
            0  0   0 0   3 0 0 1
                   3 1   3 5 0 8
                   3 1   3 1 4 5
```

```
8  4  1  7  2       7  7      0 0 1 0
3  3  2  2  2      ¯1  2      0 0 0 0
0  0  3  3  2      ¯1  7      0 0 1 0
2  0  1  1  2
8 48 67  7  2

1 1   2 2 3   2 1 5 5 5   4 0 4
0 0   2 2 3   0 5 5 9 5   4 0 4
0 0   2 2 0   3 4 1 6 5   5 7 1
0 0   0 0 0   1 1 1 0 2   3 1 3
              2 1 5 5 5

3 1 5 2   36 14 10 26 66    4 4 4
3 1 5 3    1  4  0  1  1    5 4 1
3 1 5 4    0  0  1  2  2
0 0 0 0

24  6  2  8     5 3 4
 9  6  2  8     1 1 0
                2 3 4
                3 3 2
                1 3 3
```

7.3 Item 14 should be stated as `(5+5≥9)|0-6164 1 ¯1` to avoid use of the monadic negation function first introduced in the next chapter:

```
2; 9 0 9 3 0; 3 3 1 7; 5; 3; 0; 0
0 4 0 5; 15; 56 56; 2 2 2; 90 46
9 166 17; 79; 2 2 5 6 0; 1 4 1; 0
0; 0 2; 0; 0 1 0 0; 0 0
```

```
7.4          (⍳9)∘.|¯10+⍳19
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
   0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0
   3 0 1 2 3 0 1 2 3 0 1 2 3 0 1 2 3 0 1
   1 2 3 4 0 1 2 3 4 0 1 2 3 4 0 1 2 3 4
   3 4 5 0 1 2 3 4 5 0 1 2 3 4 5 0 1 2 3
   5 6 0 1 2 3 4 5 6 0 1 2 3 4 5 6 0 1 2
   7 0 1 2 3 4 5 6 7 0 1 2 3 4 5 6 7 0 1
   0 1 2 3 4 5 6 7 8 0 1 2 3 4 5 6 7 8 0
```

Patterns in the table are similar to table 7.1

7.5 0 0 1 0 0 1 0 0 1 0 0 1 0 0
1 0 ;0 0 0 0 1 0 0 0 0 1 0 0
0 0 1 0 0 0 0 1 0 0 0 0 1;

4|*M*

0	1	2	3	0	1	2	3	0	1
2	3	0	1	2	3	0	1	2	3
0	1	2	3	0	1	2	3	0	1
2	3	0	1	2	3	0	1	2	3
0	1	2	3	0	1	2	3	0	1
2	3	0	1	2	3	0	1	2	3
0	1	2	3	0	1	2	3	0	1
2	3	0	1	2	3	0	1	2	3
0	1	2	3	0	1	2	3	0	1
2	3	0	1	2	3	0	1	2	3

9|*M*

0	1	2	3	4	5	6	7	8	0
1	2	3	4	5	6	7	8	0	1
2	3	4	5	6	7	8	0	1	2
3	4	5	6	7	8	0	1	2	3
4	5	6	7	8	0	1	2	3	4
5	6	7	8	0	1	2	3	4	5
6	7	8	0	1	2	3	4	5	6
7	8	0	1	2	3	4	5	6	7
8	0	1	2	3	4	5	6	7	8
0	1	2	3	4	5	6	7	8	0

7|*M*

0	1	2	3	4	5	6	0	1	2
3	4	5	6	0	1	2	3	4	5
6	0	1	2	3	4	5	6	0	1
2	3	4	5	6	0	1	2	3	4
5	6	0	1	2	3	4	5	6	0
1	2	3	4	5	6	0	1	2	3
4	5	6	0	1	2	3	4	5	6
0	1	2	3	4	5	6	0	1	2
3	4	5	6	0	1	2	3	4	5
6	0	1	2	3	4	5	6	0	1

7.6 0=(⍳10)∘.|⍳10

1	1	1	1	1	1	1	1	1	1
0	1	0	1	0	1	0	1	0	1
0	0	1	0	0	1	0	0	1	0
0	0	0	1	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	1

1) A number does not divide evenly into a smaller number.
2) A number divides evenly into itself.

7.7 2 5 7; 11 13 17

7.8 0=(⍳10)∘.|¯11+⍳21

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0
0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1
0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1

Yes; Column 11; Yes

7.9 12 45 87 ¯567 9876543 39 9378 345 ¯237 873; The sum of the digits of a number divisible by 3 is divisible by 3; If the sum of the digits is divisible by 3 then the number is divisible by 3; The residue of the number and the residue of the sum of the digits are both 0; Yes, for 9

7.10 25 90 1000 ¯595 98765 55 80 ¯390 240; The five residue of the number and the five residue of the last digit are 0, therefore a number ending in either 5 or 0 is divisible by 5

7.11 8 24 86 ¯456 9870 34592 162 1000 926 92; The two residue of the number and the two residue of the last digit are 0

7.13 0 1 0 1 4 3 2 1 0; 0 0 0 0 2 0 5 4 3 2 1 0; 0 1 0 3 0 3 1 7 6 5 4 3 2 1 0; 0 1 2 1 2 5 3 1 8 7 6 5 4 3 2 1 0; 0 0 0 0 4 0 3 0 6 4 2 0 11 10 9 8 7 6 5 4 3 2 1 0; 0 0 2 0 2 2 4 0 5 2 10 8 6 4 2 0 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0; 0 0 0 0 1 0 1 4 0 6 3 0 10 8 6 4 2 0 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0

7.14 1 3 9; 1 2 3 4 6 12; 1 3 5 15; 1 17; 1 2 3 4 6 8 12 24; 1 2 4 8 16 32; 1 2 3 4 6 9 12 18 36

7.15 9 3 1; 12 6 4 3 2 1; 15 5 3 1; 17 1; 24 12 8 6 4 3 2 1; 32 16 8 4 2 1; 36 18 12 9 6 4 3 2 1

7.16 Every number has an even number of factors unless it is a perfect square.

7.17 3 7 11; 5 9; 12 17 ¯4 5; 12 17 5; 12 17 5 0 0; 12 ¯4 0 ¯4 0; 12 ¯3 0 0; 12 ¯4 ¯3 0 ¯4 0; 12 0 0; ¯4 ¯4; 5 10 15 20 25; 1 6 11 16 21; 3 4 8 9 13 14 18 19 23 24; 1 1 2 1 1 2 2 2; 12 17 5 ¯3; ¯4 0 ¯4 0

7.18 a) ((0=3|⍳N)⌈(0=5|⍳N))/⍳N
or (⌈/⍉0=3 5∘.|⍳N)/⍳N

b) ((0=3|⍳N)⌊(0=5|⍳N))/⍳N
or (⌊/⍉0=3 5∘.|⍳N)/⍳N

c) (0=15|⍳N)/⍳N (This is also a solution for (b))

d) (M<⍳N)/⍳N

e) ((M<⍳N)⌊(0=5|⍳N))/⍳N

f) (⌊/⍉0=V∘.|⍳N)/⍳N
or (N=+/⍉0=V∘.|⍳N)/⍳N

g) (K=+/⍉0=V∘.|⍳N)/⍳N

7.20 4 1 25 1 11; 1100; 1; 2; 3; 4; 5; 6

7.21 0 0 0 1 0; 3 0 0 0 0; 0 2 0 0 0; 1 0 1 0 0; 0 0 0 0 1; 2 1 0 0 0; 13 is the first

7.22 0 0 0 0 0 1; 1 0 0 1 0 0; etc.; All positive integers

7.23 b) Because for any value of *I*, the quantity P[I]*A⌊B is a factor of both P[I]*A and P[I]*B.
c) No, because A⌊B is the largest power of P[I] which divides both P[I]*A and P[I]*B.

8

8.1 6; 6; 24; 24; 1 2 6 24 120 720 5040 40320 362880 3628800; 5; 6; 2 3 4 5 6 7 8 9 10 11; 2 6 24 120 720 5040 40320 362880 3628800 39916800; 1 1 2 6 24 120 720 5040 40320 362880; 1 1 2 6 24 120 720 5040 40320 362880

8.2 1; yes; undefined because of division by zero (*DOMAIN ERROR*)

8.3 ¯1 ¯2 ¯3 ¯4 ¯5 ¯6; ¯2 5 ¯3 7 ¯4; ¯1 ¯8 0 ¯10 1; ¯1 ¯8 0 ¯10 1; 0 0 0 0 0; 4 ¯10 6 ¯14 8; ¯4 ¯25 ¯9 ¯49 ¯16; 2 5 3 7 4; ¯2 ¯5 ¯3 ¯7 ¯4

8.4 .25; .2; .167; 1 .5 .333 .25 .2 .167 .143 .125; ¯1 ¯.5 ¯.333 ¯.25 ¯.2 ¯.167 ¯.143 ¯.125; Same as preceding; 1 .5 .167 .042 .008; 1.72; .5 .25 .125 .062 .031; .969

8.5 Values approach 1 and cannot exceed 1.

8.6 a) 3 4 7 9 10; 3 4 7 9 10; 3 4 7 9 10; ¯3 ¯4 ¯7 ¯9 ¯10; 1 ¯1 1 ¯1 ¯1; 33; 13; 1 0 1 0 0; 3 7; ¯4 ¯9 ¯10
b) 7.2 3.4 8.1 6; 7.2 3.4 8.1 6
c) They yield the same result for any value of *P*.

8.7 3 ¯3 2 ¯5; 4 ¯2 2 ¯4; 0 1 1
2 2 3 3 4 4 5; 1 1 2 2 3 3 4
4 5 5; 0 0 1 1 1 2 2 2 3 3; 1 1 1
2 2 2 3 3 3 4; 0 0 1 0 1; ¯6 7;
1.8 ¯2.7 4.9; 0 0 1 1 1 2 2 2 3 3
3 4; 0 0 1 1 1 2 2 2 3 3 3 4; 0 0
0 0 1 1 1 1 1 2 2 2; 0 0 0 0 1 1
1 1 1 2 2 2

8.8 0 0 1 0 1 0; 1 1 0 1 0 1; 0
0 0 1 1; 1 1 1 0 0; 1 1 1 0
0; 1 1 1 1 0 1 1 1 1 0 1 1; 1 1 1
1 0 1 1 1 1 0 1 1

8.9 The results are shown in order across the page:

```
0 0     0 0     0 1     0 1
0 1     0 1     1 1     1 1

0 1     0 1     0 1     0 1
1 0     1 0     0 0     0 0
```

8.10 c) Any pair of function related like the pairs in parts a) and b) are called <u>duals</u>. The columns of the following table show dual pairs:

```
⌈  <  =  >
⌊  ≤  ≠  ≥
```

8.11 1 2 3 2 4 6 3 6 9; 36; 6 12
18; 36; 1 2 3 4; 1 2 3 4 1
2 3 4; 1 2 3 4 1 2 3 2 4 6 3 6 9

8.12 3; 5; 3; 5; 3 5; 15; 5 3; 5
3; 15; 3; empty vector; 1

9

9.1

```
∇Z←D6 X
Z←0=6|X∇
```

1; 0 0 0 0 0 1 0 0 0 0 0 1;

```
      D6 (⍳10)∘.+(⍳10)
0 0 0 0 1 0 0 0 0 0
0 0 0 1 0 0 0 0 0 1
0 0 1 0 0 0 0 0 1 0
0 1 0 0 0 0 0 1 0 0
1 0 0 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0 0 0
0 0 0 0 1 0 0 0 0 0
0 0 0 1 0 0 0 0 0 1
0 0 1 0 0 0 0 0 1 0
0 1 0 0 0 0 0 1 0 0
```

```
      D6 (⍳10)∘.×(⍳10)
0 0 0 0 0 1 0 0 0 0
0 0 1 0 0 1 0 0 1 0
0 1 0 1 0 1 0 1 0 1
0 0 1 0 0 1 0 0 1 0
0 0 0 0 0 1 0 0 0 0
1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 1 0 0 0 0
0 0 1 0 0 1 0 0 1 0
0 1 0 1 0 1 0 1 0 1
0 0 1 0 0 1 0 0 1 0
```

```
      D6 (⍳10)∘.-(⍳10)
1 0 0 0 0 0 1 0 0 0
0 1 0 0 0 0 0 1 0 0
0 0 1 0 0 0 0 0 1 0
0 0 0 1 0 0 0 0 0 1
0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0
1 0 0 0 0 0 1 0 0 0
0 1 0 0 0 0 0 1 0 0
0 0 1 0 0 0 0 0 1 0
0 0 0 1 0 0 0 0 0 1
```

9.2

```
∇Z←B X
Z←X*2∇
 B⍳6
```

1 4 9 16 25 36;

```
      B (⍳6)∘.+(⍳6)
 4   9  16  25  36  49
 9  16  25  36  49  64
16  25  36  49  64  81
25  36  49  64  81 100
36  49  64  81 100 121
49  64  81 100 121 144
```

9.3

```
∇Z←R7 X
Z←7|X∇
```

1 2 3 4 5 6 0 1 2 3 4 5

9.4

```
∇Z←IQ7 X
Z←⌊X÷7∇
```

0 10 3 7

```
9.5   0 0 0 0 0 3 0 0 0 0; 1; 1;
      27 48 75 108 147; 14 14 14
14 21 21 21 28; same as X
```

```
9.6   a) 0 0 0 0 0 0 1 0
      b) 0 0 0 0 0 0 1 0
```

```
9.7   a) ∇Z←SQUARE S
           Z←S*2∇
```

```
b) ∇Z←CAFR R
   Z←3.1416×R*2∇
```

```
c) ∇Z←CAFD D
   Z←CAFR D÷2∇
```

```
d) ∇Z←SVFR R
   Z←(4÷3)×3.1416×R*3∇
```

```
e) ∇Z←FTOI X
   Z←12×X∇
```

```
9.8   0 0 0 1; 0 0 0 1; 0 0 0 0;

        M F 7+M
0 0 0 0 0
0 0 0 0 1
0 0 0 1 0
0 0 1 0 0
0 1 0 0 0
```

```
9.9   ∇Z←L H W
        Z←L×W∇
12; 15 24 35; 15 18 21; 15 20 25
```

```
9.10   ∇Z←H K L
         Z←H×L*2∇
```

```
9.11   a)∇Z←B AOT A
          Z←.5×B×A∇
```

```
b)∇Z←L PER W
    Z←2×L+W∇
```

```
c)∇Z←A WOR L
   Z←A÷L∇
```

```
d)∇Z←L WORR A
    Z←A÷L∇
```

```
e)∇Z←H VOCC R
   Z←H×3.1416×R*2∇
```

```
f)∇Z←A ALT B
    Z←2×A÷B∇
```

```
9.12   a) ∇Z←AREA S
            Z←S×(432÷2)-S∇
       b) 108 (a square)
```

```
9.13   a) ∇Z←L A S
            Z←S×(L÷2)-S∇
       c) S is L÷4
```

```
9.14   4; 6; 8
```

```
9.15   ¯6.111 ¯5.556 ¯5 ¯4.444
        ¯3.889 ¯3.333 ¯2.778 ¯2.222
¯1.667 ¯1.111; 21 22 23 24 25 26
27 28 29 30; 69.8 71.6 73.4 75.2
77 78.8 80.6 82.4 84.2 86; 21 22
23 24 25 26 27 28 29 30
```

```
9.16    10 8; 1.25; 1.25; 58 42;
       213 24; 277 70; 744 377
```

9.17 The function *P* mentioned in the question should have been *A*. The function *M* requested is the same as function *P*: 3 8; .375; .375; 20 42; 315 24; 27 70

```
9.18   ∇Z←Y D Y
        Z    ⌽Y∇
3 8; .375; 1.5; 30 28
```

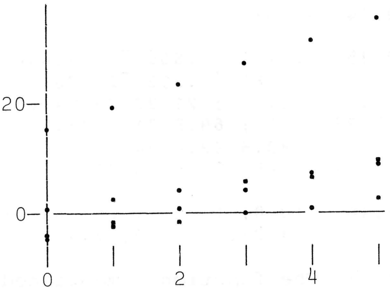
```
9.19        Q←R 3              Q←R 4
       R[1] 12            R[1] 16
       R[2] 27            R[2] 48
       R[3] 54            R[3] 128
       R[4] 93            R[4] 192

            Q←R 3              Q←R 4
       R[2] 27            R[2] 48
       R[4] 93            R[4] 192
```

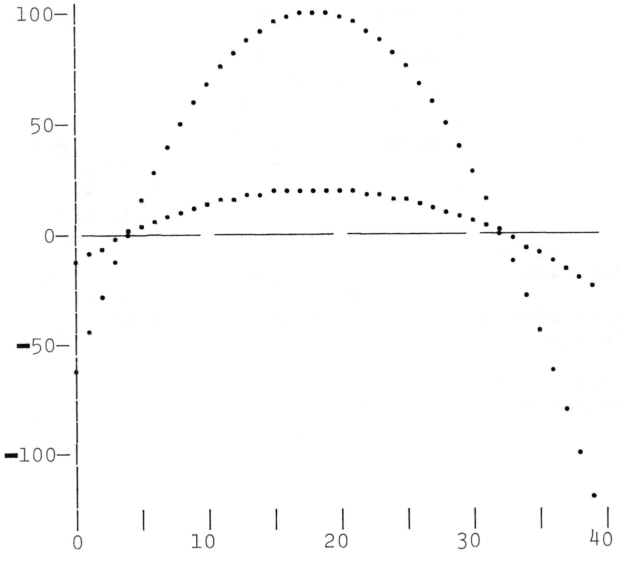
10

10.1 .4+1.7×*X*; ¯3.9+1.2×*X*; ¯4.7+2.8×*X*; 15+4×*X*

10.3



10.4



10.6 a) .4 1.7 *L X*; ¯3.9 1.2 *L X*; ¯4.7 2.8 *L X*; 15 4 *L X*

10.7 Answers are given for the first function only and are approximate:

a) Between 3 and 4 (but nearer 4) and between 32 and 33 (nearer 32)
b) 5 and 31; 2.8 and 33.2; none
c) 18
d) At both ends

10.8 The graph of the first function shows it to be nearly zero for the arguments 4 and 32, and the smallest tabled values of the function occur for those arguments. We may therefore evaluate the expression (X-4)×(X-32), which will at least have zero values at about the right place. Comparison with the original function for some argument (say for 10) shows the value of (X-4)×(X-32) to be about ¯10 times too large (¯132 as compared to 13.6). We therefore evaluate the expression ¯.1×(X-4)×(X-32) and find it a fairly good approximation. An exact expression (obtainable by the methods of Section 10.8) is ¯12.4+(3.5×X)+(¯.1×X×(X-1)).

The exact expression for the second function is
¯61+(10.41×X)+(¯.57×X×(X-1))+
  (.01×X×(X-1)×(X-2))

10.9 1 2 3; 3 4 5; 4 5; 1 2; 1 2 3 4 5 0 0; 0 0 1 2 3 4 5; 5; 3; 1 2 3; 7 9 11; 7 9 11 4 5

10.10 a) 1 4 9 16 25 36; 0 1 4 9 16 25; 1 3 5 7 9 11; 2 2 2 2 2; 0 0 0 0
b) 1 8 27 64 125 216; 0 1 8 27 64 125; 1 7 19 37 61 91; 6 12 18 24 30; 6 6 6 6
c) V is 10⍴1.8; W is 9⍴0
d) V is 3.5 3.3 3.1 2.9 2.7 etc.; W is ¯.2 ¯.2 ¯.2 ¯.2 etc.
e) V is 7⍴1; W is 6⍴0

10.11

F1	D F1	F2	D F2
.4	1.7	¯3.9	1.2
2.1	1.7	¯2.7	1.2
3.8	1.7	¯1.5	1.2
5.5	1.7	¯.3	1.2
7.2	1.7	.9	1.2
8.9		2.1	

F3	D F3	F4	D F4
¯4.7	2.8	15	4
¯1.9	2.8	19	4
.9	2.8	23	4
3.7	2.8	27	4
6.5	2.8	31	4
9.3		35	

10.15

F1	D F1	D D F1
¯12.4	3.5	¯.2
¯8.9	3.3	¯.2
¯5.6	3.1	¯.2
¯2.5	2.9	¯.2
.4	2.7	¯.2
3.1	2.5	etc
5.6	etc	
etc		

F2	D F2	D D F2	D D D F2
¯61.00	10.41	¯1.14	.06
¯50.59	9.27	¯1.08	.06
¯41.32	8.19	¯1.02	.06
¯33.13	7.17	¯.96	.06
¯25.96	6.21	¯.90	etc
¯19.75	5.31	etc	
¯14.44	etc		
etc			

10.16 See solutions to Ex. 10.8

10.17 Any further columns would consist of zeros.

10.18 (×/C)+((1-+/C)×X)+(X×X-1)

10.19 (X-C[1])×(X-C[2])×(X-C[3]) is equivalent to
S0+(S1×X)+(S2×X*2)+(S3×X*3)
for S0 equal to ×/-C
  S1 equal to (×/-C[1 2])+(×/-C[1 3])+(×/-C[2 3])
  S2 equal to +/-C
  S3 equal to 1
and is also equivalent to
T0+(T1×X)+(T2×X×(X-1))+
  (T3×X×(X-1)×(X-2))
for T0 equal to S0
  T1 equal to S1+S2+1
  T2 equal to S2+3
  T3 equal to 1
for example: if C←2 3 4 then

S0	¯24	T0	¯24
S1	26	T1	18
S2	¯9	T2	¯6
S3	1	T3	1

10.20 a)

F1	D F1	D D F1	D D D F1
¯37.2	10.5	¯.6	0
¯26.7	9.9	¯.6	0
¯16.8	9.3	¯.6	0
¯7.5	8.7	¯.6	0
1.2	8.1	¯.6	etc
9.3	7.5	etc	
16.8	etc		
etc			

10.21 a) The expressions must agree.

10.22 ×/*X*-0.⍳7

```
 Y        D Y     D D Y    D D D Y
   0 |       0 |       0 |       0 |
   0 |       0 |       0 |       0 |
   0 |       0 |       0 |       0 |
   0 |       0 |       0 |       0 | etc
   0 |       0 |       0 |  5040 |
   0 |       0 |  5040 |         |
   0 |  5040 |         |         |
5040 |       |         |         |
```

10.23 ¯3 ¯2 ¯1 0 1 2 3; 6 1 ¯2 ¯3 ¯2 1 6; 6 5 4 3 2 1 0 ¯1 ¯2 ¯3;

```
        M
1 0 0 0 0 0 1
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 1 0 0 0 1 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 1 0 1 0 0
0 0 0 1 0 0 0
```

10.25 b)

```
⎕⎕⎕⎕⎕⎕⊛
⎕⎕⎕⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕
⊛⎕⎕⎕⎕⊛⎕
⎕⎕⎕⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕
⎕⊛⎕⎕⊛⎕⎕
⎕⎕⎕⎕⎕⎕⎕
⎕⎕⊛⊛⎕⎕⎕
```

10.26 The 1's lie in:

i) 3 bands down the main diagonal

ii) triangles in lower left and upper right corners (11 columns)

iii) cases i and ii combined

iv) (1+2×7) bands down main diagonal

v) Opposite of solution iv

vi) One upward diagonal starting in column 7 of last row

vii) In the following positions:

```
row:    3 5 6 7 11 12 13 15
column: 7 6 5 3 15 13 12 11
```

viii) The overlap of *M* and ⌽*M*, where *M* is the solution to vii

10.27

```
   .01≥W         .02≥W        .1≥W
⊛⊛⎕⎕⎕⎕⎕⎕⎕⎕⎕  Result is  ⊛⊛⊛⊛⎕⎕⎕⎕⎕⎕⎕
⎕⎕⊛⎕⎕⎕⎕⎕⎕⎕⎕   same as   ⊛⊛⊛⊛⎕⎕⎕⎕⎕⎕⎕
⎕⎕⎕⊛⎕⎕⎕⎕⎕⎕⎕   for .01   ⊛⊛⊛⊛⊛⎕⎕⎕⎕⎕⎕
⎕⎕⎕⎕⊛⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⊛⊛⊛⎕⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⊛⊛⎕⎕⎕⎕⎕
⎕⎕⎕⎕⎕⊛⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⊛⊛⎕⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⊛⊛⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⊛⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⊛⊛⎕⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⊛⊛⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⊛⊛⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕⊛⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⎕⊛⎕⎕⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⎕⊛⊛⎕⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⎕⎕⊛⎕⎕
⎕⎕⎕⎕⎕⎕⎕⎕⊛⎕⎕             ⎕⎕⎕⎕⎕⎕⎕⎕⊛⎕⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⎕⎕⊛⎕⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⎕⎕⎕⊛⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⊛⎕             ⎕⎕⎕⎕⎕⎕⎕⎕⎕⊛⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⎕⎕⎕⊛⎕
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⎕⎕⎕⊛⊛
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕             ⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⊛
⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⊛             ⎕⎕⎕⎕⎕⎕⎕⎕⎕⎕⊛
```

10.28 *HIGH*; *ABCD*; *DCBA*; *DCBA*; *XXXXXX*

10.29 ⎕*⎕*⎕*⎕*⎕; ⎕ ⎕ ⎕ ⎕;

```
B[1+6|(⍳7)∘.+⍳7]
 +-×*⎕
+-×*⎕ +
-×*⎕ +-
×*⎕ +-×
*⎕ +-×*
⎕ +-×*⎕
 +-×*⎕
```

I SING OF

10.30 Listed in order across the page:

```
********     *               ****
 *******     **              ****
  ******     ***             ****
   *****     ****            ****
    ****     *****       ********
     ***     ******      ********
      **     *******     ********
       *     ********    ********

     ***     ********    ********
     ***     ********    ********
     ***     ********    ********
     ***     ***         ***  ***
     ***     ***         ***  ***
********     ***         ********
********     ***         ********
********     ***         ********
```

10.31

```
 C[M]          C[5⌊M]
 ∘-+×○⋆⎕       ∘-+××××
∘∘-+×○⋆⎕      ∘∘-+××××
---+×○⋆⎕      ---+××××
++++×○⋆⎕      ++++××××
×××××○⋆⎕      ××××××××
○○○○○○⋆⎕      ××××××××
⋆⋆⋆⋆⋆⋆⋆⎕      ××××××××
⎕⎕⎕⎕⎕⎕⎕⎕      ××××××××

 C[5⌈M]        C[M⌈⊖⌽M]
×××××○⋆⎕      ⎕⎕⎕⎕⎕⎕⎕⎕
×××××○⋆⎕      ⎕⋆⋆⋆⋆⋆⋆⎕
×××××○⋆⎕      ⎕⋆○○○○⋆⎕
×××××○⋆⎕      ⎕⋆○××○⋆⎕
×××××○⋆⎕      ⎕⋆○××○⋆⎕
○○○○○○⋆⎕      ⎕⋆○○○○⋆⎕
⋆⋆⋆⋆⋆⋆⋆⎕      ⎕⋆⋆⋆⋆⋆⋆⎕
⎕⎕⎕⎕⎕⎕⎕⎕      ⎕⎕⎕⎕⎕⎕⎕⎕
```

11

11.1 a)
```
∇Z←P X
 Z←8+4×X∇
```
b)
```
∇Z←Q X
 Z←(÷4)×¯8+X∇
```

c) ¯2 ¯1.75 ¯1.5 ¯1.25 ¯1 ¯.75; 0 1 2 3 4 5; 8 12 16 20 24 28; 0 1 2 3 4 5

11.2 a-b)

```
∇Z←F1 X             ∇Z←G1 X
Z←¯3+2×X∇           Z←(÷2)×3+X∇

∇Z←F2 X             ∇Z←G2 X
Z←¯8+10×X∇          Z←(÷10)×8+X∇

∇Z←F3 X             ∇Z←G3 X
Z←¯2+¯10×X∇         Z←(÷¯10)×2+X∇

∇Z←F4 X             ∇Z←G4 X
Z←4+3×X∇            Z←(÷3)×(-4)+X∇

∇Z←F5 X             ∇Z←G5 X
Z←4×X∇              Z←(÷4)×X∇

∇Z←F6 X             ∇Z←G6 X
Z←5+X∇              Z←(-5)+X∇
```

c) ¯7 ¯5 ¯3 ¯1 1; ¯2 ¯1 0 1 2; .5 1 1.5 2 2.5; ¯2 ¯1 0 1 2

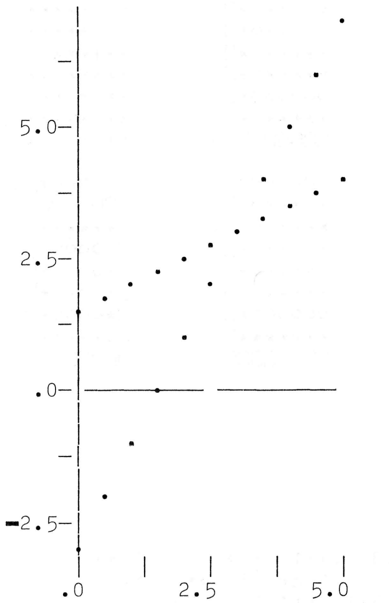
11.3
```
.4+1.7×(÷.2)×(-2)+X
   or ¯16.6+8.5×X
¯3.9+1.2×(÷.2)×(-2)+X
   or ¯15.9+6×X
¯4.7+2.8×(÷.2)×(-2)+X
   or ¯32.7+14×X
15+4×(÷.2)×(-2)+X
   or ¯25+20×X
```

11.4 Certain of the alternative expressions shown below are approximate only (e.g. .567 for 1.7÷3):

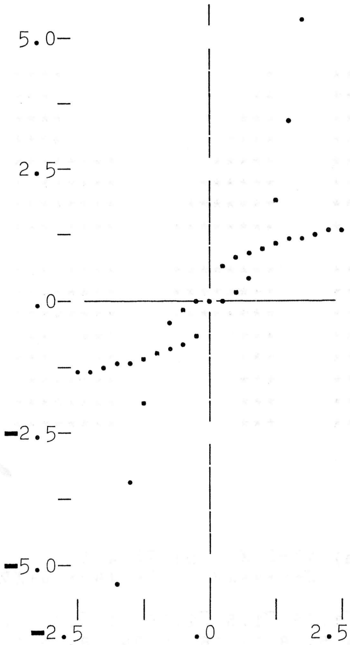
a)
```
.4+1.7×(÷3)×7+X
   or 4.367+.567×X
¯3.9+1.2×(÷3)×7+X or ¯1.1+.4×X
¯4.7+2.8×(÷3)×7+X
   or 1.833+.933×X
15+4×(÷3)×7+X or 24.333+1.333×X
```

b)
```
.4+1.7×(÷1.5)×2.5+X
   or 3.233+1.133×X
¯3.9+1.2×(÷1.5)×2.5+X
   or ¯1.9+.8×X
¯4.7+2.8×(÷1.5)×2.5+X
   or ¯.033+1.867×X
15+4×(÷1.5)×2.5+X
   or 21.667+2.667×X
```

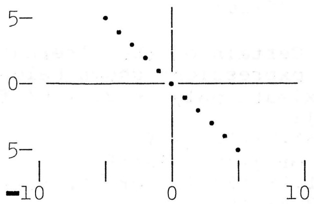
11.6 Plots for *F*1 and *G*1 only:



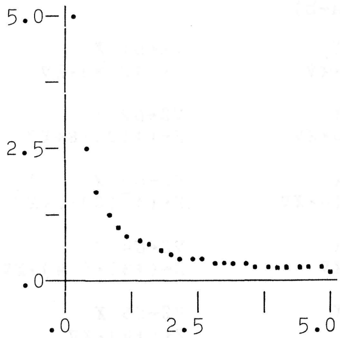
11.7



11.8 -*X* is its own inverse:



11.9 ÷*X* is its own inverse:



11.10 1.732; 2.236; 2.449; 64

11.11 1.442; 1.710; 1.817; 16

11.12 2; 1.75; 14÷3 or 4.667; 120.5; 90; 688

11.13 2.236; 1.817; 16; 20; ¯8

12

12.1

```
      SQRT 5
SQRT[1] 1
SQRT[2] 3
SQRT[3] →2
SQRT[2] 2.3333333
SQRT[3] →2
SQRT[2] 2.2380952
SQRT[3] →2
SQRT[2] 2.2360689
SQRT[3] →2
```

```
      SQRT 25
SQRT[1] 1
SQRT[2] 13
SQRT[3] →2
SQRT[2] 7.4615385
SQRT[3] →2
SQRT[2] 5.406027
SQRT[3] →2
SQRT[2] 5.0152476
SQRT[3] →2
```

```
      SQRT 0.25
SQRT[1] 1
SQRT[2] 0.625
SQRT[3] →2
SQRT[2] 0.5125
SQRT[3] →2
SQRT[2] 0.5001524
SQRT[3] →2
SQRT[2] 0.5
SQRT[3] →2
```

12.2

```
      SQT 5
SQT[1] 1
SQT[2] 3
SQT[3] →2
SQT[2] 2.3333333
SQT[3] →2
SQT[2] 2.2380952
SQT[3] →2
SQT[2] 2.2360689
SQT[3] →0
2.2360689
```

```
      SQT 25
SQT[1] 1
SQT[2] 13
SQT[3] →2
SQT[2] 7.4615385
SQT[3] →2
SQT[2] 5.406027
SQT[3] →2
SQT[2] 5.0152476
SQT[3] →2
SQT[2] 5.0000232
SQT[3] →2
SQT[2] 5
SQT[3] →0
5
```

```
      SQT 0.25
SQT[1] 1
SQT[2] 0.625
SQT[3] →2
SQT[2] 0.5125
SQT[3] →2
SQT[2] 0.5001524
SQT[3] →2
SQT[2] 0.5
SQT[3] →0
0.5
```

12.3
12.4 In the following the complete trace is shown only for the first two iterations and thereafter only the significant line [1] is shown:

```
      4 5 GRF 20
GRF[1] 4.5
GRF[2] 4.5
GRF[3] →1
GRF[1] 4.25
GRF[2] 4.25
GRF[3] →1
GRF[1] 4.375
GRF[1] 4.4375
GRF[1] 4.46875
GRF[1] 4.484375
GRF[1] 4.4765625
GRF[1] 4.4726563
GRF[1] 4.4707031
GRF[1] 4.4716797
GRF[1] 4.472168
GRF[1] 4.4719238
GRF[1] 4.4720459
GRF[1] 4.4721069
GRF[1] 4.4721375
GRF[1] 4.4721222
```

```
      3 2 GRF 3
GRF[1] 2.5
GRF[2] 2.5
GRF[3] →1
GRF[1] 2.25
GRF[2] 2.25
GRF[3] →1
GRF[1] 2.125
GRF[1] 2.1875
GRF[1] 2.21875
GRF[1] 2.234375
GRF[1] 2.2265625
GRF[1] 2.2304688
GRF[1] 2.2324219
GRF[1] 2.2333984
GRF[1] 2.2338867
GRF[1] 2.2341309
GRF[1] 2.2340088
GRF[1] 2.2340698
GRF[1] 2.2341003
GRF[1] 2.2340851
```

```
GRF[1] 4.4721298                    GRF[1] 2.2340927
GRF[1] 4.4721336                    GRF[1] 2.2340889
GRF[1] 4.4721355                    2.2340889
4.4721355
```

12.5
```
∇Z←F X
Z←X*4∇
2 3 GRF 17
```

12.6

```
∇Z←F X            ∇Z←F X         ∇Z←F X           ∇Z←F X
Z←(X-2)*3∇        Z←X*5∇         Z←(3+2×X)*2∇     Z←(¯2+.5×X)*6∇
5 6 GRF 29        3 4 GRF 265    .5 1 GRF 19      7 8 GRF 47
```

12.7

```
  GCD 35 133        GCD 133 35       GCD 140 35        GCD 1728 840
GCD[1] 35         GCD[1] 133       GCD[1] 140        GCD[1] 1728
GCD[2] 28 35      GCD[2] 35 133    GCD[2] 35 140     GCD[2] 840 1728
GCD[3] →1         GCD[3] →1        GCD[3] →1         GCD[3] →1
GCD[1] 28         GCD[1] 35        GCD[1] 35         GCD[1] 840
GCD[2] 7 28       GCD[2] 28 35     GCD[2] 0 35       GCD[2] 48 840
GCD[3] →1         GCD[3] →1        GCD[3] →0         GCD[3] →1
GCD[1] 7          GCD[1] 28        35                GCD[1] 48
GCD[2] 0 7        GCD[2] 7 28                        GCD[2] 24 48
GCD[3] →0         GCD[3] →1                          GCD[3] →1
7                 GCD[1] 7                           GCD[1] 24
                  GCD[2] 0 7                         GCD[2] 0 24
                  GCD[3] →0                          GCD[3] →0
                  7                                  24
```

12.8 a) 3 4; 5 19; 9 53; 7 81; 8 1

b) *V* and *V÷GCD V* represent the same rational number.

12.9 a) 10 8; 475 900; 88 128; 5000 5000

b) 2; 25; 8; 5000

12.10 a)
```
∇Z←X PLUS Y
 Z←(X A Y)÷GCD X A Y∇
```

b)
```
     ∇Z←X PLUS2 Y
[1]  Y←(+/X×⌽Y),X[2]×Y[2]
[2]  X←Y
[3]  Z←X[1]
[4]  X←(|/X),X[1]
[5]  → 3 6[1+0=X[1]]
[6]  Z←Y÷X∇
```

12.11
```
∇Z←A TIMES B
 Z←(A×B)÷GCD A×B∇
```

12.12 1; 2; 4; 8; 16; 32; 64; 128; 2*N; 4096

12.13 1; 0; 0; 0; 0; 0; 0; 0

12.14 *X*Y*; *!X*; *X⌈Y*

12.15
```
∇Z←D N
 Z←1≠N∇
```

(Other solutions are possible and should be checked by evaluation)

12.16
```
∇Z←X MIN Y
 Z←-(-X)⌈(-Y)∇
```

```
∇Z←MAG X                  ∇Z←X NEQ Y
 Z←X⌈-X∇                   Z←~X=Y∇
```

12.17 a)
```
∇Z←X RES Y
[1] Z←Y
[2] →3×X≤Z
[3] Z←Z-X
[4] →2∇
```

b)
```
∇Z←X GRES Y
[1] Z←Y
[2] →3×(X≤Z)⌈(Z<0)
[3] Z←Z+(X×Z<0)-(X×Z>0)
[4] →2∇
```

12.18

a)
```
 ∇Z←FLOOR X
[1] Z←-⌈-X∇
```

b)
```
 ∇Z←FLO X
[1] Z←0
[2] →3×X≥1
[3] Z←Z+1
[4] X←X-1
[5] →2∇
```

c)
```
 ∇Z←FLON X
[1] Z←0
[2] →3×(X≥1)⌈(X<0)
[3] Z←Z+(X>0)-(X<0)
[4] X←X-(X>0)-(X<0)
[5] →2∇
```

12.19 *W N* generates the primes up to and including *N*. The symbol ∨ used on line 5 of *W* is (as stated in the Summary of Notation) the or function defined in Section 14.2 and is equivalent to the function ⌈ when appied to logical arguments.

13

13.1 35; 21; 3; 3; 0; 1; 0; 3; 65; 65; 11; 2; ¯6; 6; 0; 3; 0; 5

13.2 The number of positions in which the elements in *P* equal the corresponding elements of *Q*; tells if all of the elements of *P* do not equal the corresponding elements of *Q*; tells if any of the elements of *P* equal the corresponding elements in *Q*; gives the product over the sum of *P* and *Q*; gives the largest among the element-by-element sum of *P* and *Q*

13.3 `A+.×B; A+.⌈B; A⌊.⌈B; A+.≤B; A⌊.≤B; A⌈.≤B; A×.-B; A+.|B; A+.*B; B+.*A; C+.×D; C⌈.⌊D; C⌊.⌈D; (|C)⌈.⌊(|D); (|C)⌊.⌊(|D); C+.≤D; C+.=D; C+.-D;`

13.4 1100; 210; 10; 23100; 1100; 8; ¯1; ¯1; 0; 10; 23; ¯18; 0; 1; 0; 1

13.5 216; 216; 216; 216; 36; 36; 1296; 1296; 64; 64; 512; 512; 4096

13.6 1 2 3; 3; 1 2 3 4 5 6 1 2; 8; 1 4 2 4 2 4 1; 1 0 1 0 1 0 1

13.7

```
 M          N                4 3ρι12
1 2 3      1 2 3 4 5       1  2  3
4 5 6      6 1 2 3 4       4  5  6
           5 6 1 2 3       7  8  9
                          10 11 12

 ρM            ρN
2 3           3 5

 ⍉3 4ρι12          ρ4 3ρι12
1  5  9           4 3
2  6 10
3  7 11            ρ⍉4 3ρι12
4  8 12           3 4
```

13.8 In order down the page:

```
 1  1  3  1        0 0 1 0
 2  3  3  3        0 0 0 3

¯9 ¯5  0 ¯6        0 0 1 0
¯3 ¯6 ¯1 ¯8        0 0 0 3

¯3 ¯8  1 ¯11       1 1 3 1
¯8 ¯5 ¯5  ¯5       2 3 3 3

29 ¯12  ¯7 ¯16     2 2 1 2
¯6   8 ¯17  17     1 0 0 3

29 ¯12  ¯7 ¯16     1 1 2 1
¯6   8 ¯17  17     2 3 3 0
```

13.9 The result is a matrix *R* such that if *A* is the *I*th row vector of *M* (that is, `A←M[I;]`) and *B* is the *J*th column of *N*, then the element `R[I;J]` is determined as follows:

- The number of elements in which *A* is dominated by *B*
- The minimum of the element-by-element sum of *A* and *B*
- The sum over the minimum of *A* and *B*
- The sum over the products of *A* and *B*
- Same as the previous case
- The number of elements in which *A* equals *B*

13.10 1 4 16 64 256; 1 4 16 64 256; 1 5 25 125 625; 1 5 25 125 625; 1 8 64 512 4096; 1 8 64 512 4096

13.11

```
              2 5 10 17 28;
     M        2 5 10 17 28;
1 1 1 1 1     28 26 23 18 11;
0 1 1 1 1     2 5 10 17 28;
0 0 1 1 1     2 6 30 210 2310;
0 0 0 1 1     2 6 30 210 2310;
0 0 0 0 1     2310 1155 385 77 11
```

13.12 2 4 8 16 32; 2 4 8 16 32; 14 3 16 ¯7 0; 2 2 4 8 16; 14 ¯11 13 ¯23 7

13.13 a) *I*←(⍳5)∘.=⍳5
b) *D*←*I*-(⍳5)∘.=1+⍳5
c) *I*←(⍳*N*)∘.=⍳*N*
D←*I*-(⍳*N*)∘.=1+⍳*N*
d) *X*
e) The first element of *X* followed by the differences between successive pairs of elements of *X*
f) Delete the first row of *D*
g) *D*1←((1↓⍳*N*)∘.=⍳*N*)-(1↓⍳*N*)∘.=1+⍳*N*

13.14 a) The matrices *S*+.×*D* and *D*+.×*S* are equal and one is shown at the left. The other results are listed in order:

```
1 0 0 0 0        1 3 5 7 9;
0 1 0 0 0        1 4 9 16 25;
0 0 1 0 0        1 4 9 16 25;
0 0 0 1 0        1 5 14 20 55;
0 0 0 0 1        1 4 9 16 25;
                 1 4 9 16 25
```

b) They are inverse functions.

13.15 a) 1100 21000 450 110000; 50; 22 420 9 2200

13.16 a) 1100 21000 450 110000; 6930000; 6300 330 15400 63

13.17 b) ¯8 7 12; ¯8 7 12; ¯23 2 22; ¯23 2 22; *A* and *B* are equal; *C* and *D* are equal

13.18 *C*←(*V*[1]×*M*[1;])+(*V*[2]×*M*[2;])+(*V*[3]×*M*[3;])
D←*V*+.×*M*

13.19 a) 1 4 16 64; 5 8 0 64; 77; 5; 20; 20; 112; 112
b) Reading from lowest to highest at the extreme right, the curves are: the polynomial followed by the terms in increasing order

13.20 2 5 0 500; 507

13.21

```
  C TERMS X           SUM C TERMS X
1  12  48  64             125
0   0   0 125             125
1  15  75 125             216
0   0   0 216             216
1   0   0   0               1
0   0   0   1               1
1   8  24  32  16          81
0   0   0   0  81          81
```

13.22 5 14 49 122 245 430; ¯5 ¯10 ¯17; ¯245 ¯122 ¯49; 1 2 3 4 5 6; 1 2 3 4 5 6; 1 4 9 16 25 36; 1 4 9 16 25 36; 1 8 27 64 125 216; 1 8 27 64 125 216

13.23 5 10 37 104 229 430; 5 27 67 125 201; 22 40 58 76; 18 18 18; 5 11 53 185 485 1055 2021 3533; 24 24 24 24

13.24 1 3 6 10 15; 1 3 6 10 15; 1 5 14 30 55; 1 5 14 30 55; 1 9 36 100 225; 1 9 36 100 225; 1 2 3 4 5; 1 2 3 4 5

14

14.1 a)

```
+ - × ÷ < ≤ = | ⌈ ⌊
1 0 1 0 0 0 1 0 1 1
```

b) 3 *F* 5 does not equal 5 *F* 3 where *F* is any one of the functions associated with a zero in part a)

14.2

```
    ∇Z←COM1 X
[1] C←'+-×÷<≤=|⌈⌊'
[2] N←1 0 1 0 0 0 1 0 1 1
[3] Z←(X=C)/N∇
```

14.3 a)

```
      A←0 1∘.∧0 1              B←0 1∘.∨0 1
      A=⍉A                     B=⍉B
1 1                      1 1
1 1                      1 1
      ∧/,A=⍉A                  ∧/,B=⍉B
1                        1
```

b) 0; 1; 1; 0; 0 0 0 1; 0 1 1 1; 1 1 1 0; 1 0 0 0

14.6

X	Y	Z	X∧Y	(X∧Y)∧Z	Y∧Z	X∧(Y∧Z)
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

The function <u>and</u> is associative because the columns for (X∧Y)∧Z and X∧(Y∧Z) agree.

14.7 <u>or</u> is associative but <u>nand</u> (not-and) and <u>nor</u> (not-or) are not associative.

14.8 a) `2+(3×4)` yields `14` but `(2+3)×(2+4)` yields `30`

b) `2+(3+4)` yields `9` but `(2+3)+(2+4)` yields `11`

e)

	+	×
+	0	0
×	1	0

14.9

	+	×	-	⌈	⌊
+	0	0	0	1	1
×	1	0	1	0	0
-	0	0	0	0	0
⌈	0	0	0	1	1
⌊	0	0	0	1	1

14.10 a)

X	Y	Z	Y∧Z	X∨(Y∧Z)	X∨Y	X∨Z	(X∨Y)∧ (X∨Z)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

b) ∨ distributes over ∨

c) ∧ distributes over ∧

14.11

	∨	∧
∨	1	1
∧	1	1

14.12

	∨	∧	⍲	⍱
∨	1	1	0	0
∧	1	1	0	0
⍲	0	0	0	0
⍱	0	0	0	0

14.13 b) Subtraction does not distribute over maximum.

14.16

```
      (3+14)×¯8                (¯3+5)×7
¯136                     14
      ¯8×(3+14)                7×(¯3+5)
¯136                     14
      (¯8×3)+(¯8×14)           (7×¯3)+(7×5)
¯136                     14
      (3×¯8)+(14×¯8)           (¯3×7)+(5×7)
¯136                     14
```

14.17 a)

(P⌊Q)⌈R	
R⌈(P⌊Q)	Commutativity of ⌈
(R⌈P)⌊(R⌈Q)	Distrib. of ⌈ over ⌊
(R⌈P)⌊(Q⌈R)	Commutativity of ⌈

b) For P, Q, and R equal to ¯3, 2 and 5 respectively, each line below has the result 5:

```
      (¯3⌊2)⌈5
      5⌈(¯3⌊2)
      (5⌈¯3)⌊(5⌈2)
      (5⌈¯3)⌊(2⌈5)
```

14.18 a)

```
A∧(B∧C)
(A∧B)∧C          Associativity of ∧
C∧(A∧B)          Commutativity of ∧
C∧(B∧A)          Commutativity of ∧
```

b) Same as part a) with + for ∧

c)

```
A×B×C×D
(A×B)×(C×D)      Associativity of ×
(C×D)×(A×B)      Commutativity of ×
(D×C)×(B×A)      Commutativity of ×
D×C×B×A          Associativity of ×
```

14.20 For A, B, C, and D equal to ¯1, 2, 4, and 1 respectively, each line has a result of 5:

```
(¯1+2)×(4+1)
((¯1+2)×4)+((¯1+2)×1)
(4×(¯1+2))+(1×(¯1+2))
((4×¯1)+(4×2))+((1×¯1)+(1×2))
((¯1×4)+(2×4))+((¯1×1)+(2×1))
(¯1×4)+((2×4)+(¯1×1))+(2×1)
(¯1×4)+((¯1×1)+(2×4))+(2×1)
(¯1×4)+(¯1×1)+(2×4)+(2×1)
```

14.21 a) Same as last proof on page 159 of text with ⌊ for + and + for ×

b)

```
A∧(B∨C∨D)
A∧(B∨(C∨D))                 A∨
(A∧B)∨(A∧(C∨D))            ∧D∨
(A∧B)∨((A∧C)∨(A∧D))        ∧D∨
(A∧B)∨(A∧C)∨(A∧D)           A∨
```

14.22 a) C←4 5 1

14.23 4 5 1; 4 ¯5 1; 1 2 1; 0 1 1; 0 1 1; 1 ¯2 1; 1 ¯2 1; 15 8 1; 15 ¯8 1; 15 ¯8 1

14.24 For A←3 2 ¯1 and B←4 0 2 and C←¯3 5 3 each line below has the result ¯21 10 3:

```
(3 2 ¯1+ 4 0 2)×¯3 5 3
¯3 5 3×(3 2 ¯1+4 0 2)
(¯3 5 3×3 2 ¯1)+(¯3 5 3×4 0 2)
(3 2 ¯1×¯3 5 3)+(4 0 2×¯3 5 3)
```

14.26 65; 65; 43; 43; ¯8; ¯8; 0; 0; ¯17; 57; 1; 1

14.27 25; 25; 69120; 69120; 15; 15; 23; 23

15

15.1 a) 5 ¯1 9; 3 15 12 ¯2; 2 0 4 10
b) 43 15 5 13 39; 37 2 3 28 65; ¯62 ¯4 2 16 98

15.2 a) 9 1 ¯2 8 2; 4 1 4 ¯2 4
b) ¯33 0 9 18 99; 98 13 4 11 70

15.3 a) 6 ¯2 20 ¯1 15 4; 6 ¯2 20 ¯1 15 4; 0 3 ¯5 2; 0 0 3 ¯5 2; 9 ¯15 6
b) 442; 442; 2; 4; 3

15.4 a) 2 2 4 5 1; 1 2 1; 1 3 3 1; 1 4 6 4 1; 1 5 10 10 5 1
b) 78; 9; 27; 81; 243

15.6 a) 6 5 1; 28 11 1; 28 11 1; 28 ¯11 1; 28 ¯11 1; 28 ¯11 1; 24 26 9 1; 24 26 9 1; 24 26 9 1; 0 ¯1 1; 0 2 ¯3 1; 0 ¯6 11 ¯6 1
b) 56; 108; 108; ¯2; ¯2; ¯2; 504; 504; 504; 20; 60; 120

15.7 Error in text: Q1[6] should be →4×I≠0. Answers are for the corrected version of QA.

```
      QA 2 1 3
QA[1] 1
QA[2] 2 1 3
QA[3] 3
QA[4] 3 1
QA[5] 2
QA[6] →4
QA[4] 3 4 1
QA[5] 1
QA[6] →4
QA[4] 6 11 6 1
QA[5] 0
QA[6] →0
6 11 6 1
```

```
      QA 1 1 1 1
QA[1] 1
QA[2] 1 1 1 1
QA[3] 4
QA[4] 1 1
QA[5] 3
QA[6] →4
QA[4] 1 2 1
QA[5] 2
QA[6] →4
QA[4] 1 3 3 1
QA[5] 1
QA[6] →4
QA[4] 1 4 6 4 1
QA[5] 0
QA[6] →0
1 4 6 4 1
```

15.8 a) 1 1; 1 2 1; 1 3 3 1; 1 4 6 4 1; 1 5 10 10 5 1; 1 6 15 20 15 6 1

b) They are the same.

15.10 a) 6 2 23 14 23 12

b) They are the same.

15.11 The function *Q* referred to should have been *QA* of Section 15.4

15.12

```
      QA 4⍴1                BIN 4
QA[1] 1               BIN[1] 1
QA[2] 1 1 1 1         BIN[2] →3
QA[3] 4               BIN[3] 1 1
QA[4] 1 1             BIN[4] →2
QA[5] 3               BIN[2] →3
QA[6] →4              BIN[3] 1 2 1
QA[4] 1 2 1           BIN[4] →2
QA[5] 2               BIN[2] →3
QA[6] →4              BIN[3] 1 3 3 1
QA[4] 1 3 3 1         BIN[4] →2
QA[5] 1               BIN[2] →3
QA[6] →4              BIN[3] 1 4 6 4 1
QA[4] 1 4 6 4 1       BIN[4] →2
QA[5] 0               BIN[2] →0
QA[6] →0              1 4 6 4 1
1 4 6 4 1
```

15.13 a) Column *I* corresponds to the factorial polynomial of degree *I*-1

b) 0 .167 .5 .333

c) The expression *M*+.×*V* yields a weighted sum of the columns of *M*, that is *V*[1] times the first column plus *V*[2] times the second, etc.

d) They are equivalent.

e) Since *V* is the vector derived (by the method of Section 10.6) from the difference table for the function +/(⍳*X*)*2, then *V*[1] times the 0-degree factorial polynomial plus *V*[2] times the 1-degree factorial polynomial, etc., is equivalent to +/(⍳*X*)*2. But, according to part (c), the expression *M*+.×*V* yields the coefficients of an ordinary polynomial which is equivalent to the stated weighted sum of factorial polynomials. Therefore (*M*+.×*V*) *P X* is equivalent to +/(⍳*X*)*2.

15.14 a)

```
        M
1   0   0   0   0
0   1  ¯1   2  ¯6
0   0   1  ¯3  11
0   0   0   1  ¯6
0   0   0   0   1
```

b)

```
         X: 0 1 2  3   4   5
+/(⍳X)*3: 0 1 9 36 100 225
```

c) First row is: 0 1 7 12 6

d) 0 0 .25 .5 .25

e) They are equivalent

15.15

```
∇Z←F X             ∇Z←G X                              ∇Z←C P X
Z←+/(⍳X)*2∇        Z←(+/0 1 3 2×X*0 1 2 3)÷6∇          Z←(X∘.*¯1+⍳⍴,C)+.×C∇
```

```
(F K+1)-G K+1
(+/(⍳K+1)*2)-(0 1 3 2 P K+1)÷6                        Def. of F, G, and P
((+/(⍳K+1)*2)-((0 1 3 2÷6) P K+1)                      (C P X)÷6is(C÷6)P X
((+/(⍳K)*2)+(K+1)*2)-((0 1 3 2÷6)P K+1)                +/Vis(+/¯1↓V)+¯1↑V
((F K)+(K+1)*2)-((0 1 3 2÷6) P K+1)                          Def. of F
((F K)+(0 0 1 0 P K+1))-((0 1 3 2÷6) P K+1)                  Def. of P
((F K)+((0 0 6 0÷6)P K+1))-((0 1 3 2÷6)P K+1)                (6×C)÷6 is C
(F K)+(((0 0 6 0÷6)P K+1)-((0 1 3 2÷6)P K+1))                (L+M)-N is
                                                               L+(M-N)
(F K)+((0 ¯1 3 ¯2÷6) P K+1)                  (C P X)-(D P X) is (C-D)P X
(F K)-(0 1 ¯3 2÷6) P K+1                     L+(C P X) is L-((-C)P X)
(F K)-(0 1 3 2÷6) P K                        Note 1
(F K)-(0 1 3 2 P K)÷6                        (C÷6) P X is (C P X)÷6
(F K)-(G K)                                  Def. of G and P
```

Also *F* 0 equals *G* 0 and therefore *F X* equals *G X* for *X* equal to 1, 2, 3, etc.

Note 1: *C P K*+1 equals (*B*+.×*C*)*P K* where *B* is the matrix:

```
1 1 1 1
0 1 2 3
0 0 1 3
0 0 0 1
```

When applied to the vector 0 1 ¯3 2÷6 the expression *B*+.×*C* yields the result 0 1 3 2÷6 used in the proof above. The reasoning used to show that *C P K*+1 equals (*B*+.×*C*) *P*·*K* is similar to that used in Exercise 15.13, except that *C P K*+1 is a sum of terms of the form *C*[*I*]×(*K*+1)⋆*I*, and the coefficients of the polynomial equivalent to (*K*+1)⋆*I* is the set of binomial coefficients of order *I*; these binomial coefficients appear as the columns of *B*. Anyone familiar with methods of "expanding" or "multipying out" the polynomial *C P K*+1 without the use of matrix methods, should verify that the matrix method used here is, in effect, a convenient way to organize the many calculations.

16

16.1 a) 18; 10010; 22; 1 2 0 0 0 0 0; *BI*; *BN*; *PNZZ*; 1 8; 1 0 0 1 0

b) 19; 10011; 23; 0 0 0 0 0 0 0 1; *BJ*; *BU*; *PNZP*; 1 9; 1 0 0 1 1; For the Prime Factors another column (for the prime number 19) had to be added.

c) Ordinary decimal (base ten) system; Base two system (discussed in Section 16.4); Base eight (Section 16.5); The Prime Factors system is based on work in Section 7.6 and exercises 7.20-24; R1 is the base ten system with the letters *A* to *J* substituted for the digits 0 to 9; R2 is base six with a letter substitution; R3 is discussed in Section 16.5; R4 is a vector decimal system in which the successive digits of the ordinary decimal system appear as the successive elements of a vector; R5 is a vector binary system.

d) No

16.3 There is an error in the specification of the alphabet *A*. There should have been a space between the *Z* and the quote, making the last character a space. The answers given below are with the corrected version of *A*.
3; 6; 6; 6 1 2 6 3; 2 3 5 6 6; 2 1 5 3 8 8 7; 14 15 23 27 9 19 27 20 8 5 27 20 9 13 5; *NOW IS THE TIME*

16.4 a) 7 1; 9 9; 1 2 3 4
b) *HB*; *JJ*; *BCDE*

16.5 a) These answers are the shortest possible (zeros can be added to the right of any one of them):
5 1 1; 9; 6 1 1; 0 1 0 0 0 0 0 0 0 0 0 1; 1 preceded by 33 zeros (that is, (33⍴0),1); 0 0 3
b) 480; 512; 960; 125; 210; 290

16.6 60; 15; 1; 49

16.7 a) 4 1 1 1; 6 1 1; 5 1 1 0 1; 0 1 1 2 1; 0 1 1 0 1 0 0 0 0 0 1
b) 1680; 960; 5280; 8085; 5115

16.8 a) 3 2 1; 0 1 2 1; 0 0 1 0 1 0 1; 4 4 0 2; 4 1 3 0 1
b) 360; 525; 935; 63504; 66000

16.9 a) 11; 40; 17

b)
```
∇Z←V PFDIV W
 Z←∧/V≥W∇
```

16.10

```
    IVDVAL 548              IVDVAL 176
IVDVAL[1]               IVDVAL[1]
IVDVAL[2] 8             IVDVAL[2] 6
IVDVAL[3] 54            IVDVAL[3] 17
IVDVAL[4] →2            IVDVAL[4] →2
IVDVAL[2] 4 8           IVDVAL[2] 7 6
IVDVAL[3] 5             IVDVAL[3] 1
IVDVAL[4] →2            IVDVAL[4] →2
IVDVAL[2] 5 4 8         IVDVAL[2] 1 7 6
IVDVAL[3] 0             IVDVAL[3] 0
IVDVAL[4] →0            IVDVAL[4] →0
5 4 8                    1 7 6
```

16.11 a) 3 7 9; 9 12 10; 7 9 8 9; 13 2 5 6; 5 9 9 10

b) first and third

c) 379; 1030; 7989; 13256; 6000

16.12

```
      2 3 5 VDADD 1 4 4
VDADD[1] 3 7 9
VDADD[2] 0 0 0
VDADD[3] →0
3 7 9

      2 3 5 VDADD 7 9 5
VDADD[1] 9 2 0
VDADD[2] 1 1 0
VDADD[3] →4
VDADD[4] 9 2 0
VDADD[5] 1 1 0
VDADD[6] →1
VDADD[1] 0 3 0
VDADD[2] 0 0 0
VDADD[3] →0
0 3 0
```

The preceding trace reveals that *VDADD* does ~~not~~ give the true sum if there is a carry from the highest order place, for this carry is "lost". *VDADD* can be modified in various ways to correct this, e.g., insert the following two lines at the beginning: *A*←0,*A* and *B*←0,*B*. Unwanted leading zeros introduced by this trick can be eliminated, e.g., by adding the line *Z*←(¯1+(0≠*Z*)⍳1)↓*Z* to be executed just before completion.

16.13 For brevity the name *SAD* is used below instead of *SERIALDADD*:

```
2 3 5 SAD 1 4 4         2 3 5 SAD 7 9 5
SAD[1]                  SAD[1]
SAD[2] 0                SAD[2] 0
SAD[3] 4                SAD[3] 4
SAD[4] 3                SAD[4] 3
SAD[5] →6               SAD[5] →6
SAD[6] 9                SAD[6] 10
SAD[7] 9                SAD[7] 0
SAD[8] 0                SAD[8] 1
SAD[9] →4               SAD[9] →4
SAD[4] 2                SAD[4] 2
SAD[5] →6               SAD[5] →6
SAD[6] 7                SAD[6] 13
SAD[7] 7 9              SAD[7] 3 0
SAD[8] 0                SAD[8] 1
SAD[9] →4               SAD[9] →4
SAD[4] 1                SAD[4] 1
SAD[5] →6               SAD[5] →6
SAD[6] 3                SAD[6] 10
SAD[7] 3 7 9            SAD[7] 0 3 0
SAD[8] 0                SAD[8] 1
SAD[9] →4               SAD[9] →4
SAD[4] 0                SAD[4] 0
SAD[5] →0               SAD[5] →0
3 7 9                   0 3 0
```

The remarks about *VDADD* in the answer to Exercise 16.2 apply equally to *SERIALDADD*.

16.14 6; 13; 15; 16

16.15

```
      IVBVAL 6                IVBVAL 13
IVBVAL[1]               IVBVAL[1]
IVBVAL[2] 0             IVBVAL[2] 1
IVBVAL[3] 3             IVBVAL[3] 6
IVBVAL[4] →2            IVBVAL[4] →2
IVBVAL[2] 1 0           IVBVAL[2] 0 1
IVBVAL[3] 1             IVBVAL[3] 3
IVBVAL[4] →2            IVBVAL[4] →2
IVBVAL[2] 1 1 0         IVBVAL[2] 1 0 1
IVBVAL[3] 0             IVBVAL[3] 1
IVBVAL[4] →0            IVBVAL[4] →2
1 1 0                   IVBVAL[2] 1 1 0 1
                        IVBVAL[3] 0
                        IVBVAL[4] →0
                        1 1 0 1
```

16.16 a)

```
0 0 0
0 0 1
0 1 0
0 1 1
1 0 0
1 0 1
1 1 0
1 1 1
```

b)

```
0 0 0 0
0 0 0 1
0 0 1 0
0 0 1 1
0 1 0 0
0 1 0 1
0 1 1 0
0 1 1 1
1 0 0 0
1 0 0 1
1 0 1 0
1 0 1 1
1 1 0 0
1 1 0 1
1 1 1 0
1 1 1 1
```

c) Formed by part b) with a column of zeros appended on the left followed by part b) with a column of 1's appended on the left.

d) Formed from c) as c) is from b).

16.17 a) *VBADD* is obtained from *VDADD* by replacing each occurrence of 10 by 2

b) 1 1 1 0 0; 1 0 0 0 0

16.18 a) *SERIALBADD* gotten from *SERIALDADD* by replacing each occurrence of 10 by 2

b) 1 1 1 0 0; 1 0 0 0 0

16.20

```
     ∇Z←A BT B
[1]   A←0,A
[2]   B←0,B
[3]   Z←B
[4]   → 5 10[1+∧/0=A]
[5]   R←A+B
[6]   A←(2=R)-(¯2=R)
[7]   B←R-3×A
[8]   A←(1↓A),0
[9]   →3
[10]  Z←(¯1+(0≠Z)⍳1)↓Z∇
```

16.21

```
    ∇Z←A N
[1]  Z←¯1+⍳N
[2]  Z←Z∘.+Z∇

    ∇Z←S N
[1]  Z←N|A N∇

    ∇Z←C N
[1]  Z←N≤A N∇
```

a)

```
 S 3        C 3
0 1 2      0 0 0
1 2 0      0 0 1
2 0 1      0 1 1
```

b)

```
  S 4          C 4
0 1 2 3      0 0 0 0
1 2 3 0      0 0 0 1
2 3 0 1      0 0 1 1
3 0 1 2      0 1 1 1
```

c)

```
   S 5            C 5
0 1 2 3 4      0 0 0 0 0
1 2 3 4 0      0 0 0 0 1
2 3 4 0 1      0 0 0 1 1
3 4 0 1 2      0 0 1 1 1
4 0 1 2 3      0 1 1 1 1
```

d)

```
     S 7                C 7
0 1 2 3 4 5 6      0 0 0 0 0 0 0
1 2 3 4 5 6 0      0 0 0 0 0 0 1
2 3 4 5 6 0 1      0 0 0 0 0 1 1
3 4 5 6 0 1 2      0 0 0 0 1 1 1
4 5 6 0 1 2 3      0 0 0 1 1 1 1
5 6 0 1 2 3 4      0 0 1 1 1 1 1
6 0 1 2 3 4 5      0 1 1 1 1 1 1
```

e) *BT*←¯2+*A* 3 and *DIG*←*BT*-3×*CAR* and *CAR*←(2=*BT*)-(¯2=*BT*)

```
    BT            DIG           CAR
¯2 ¯1  0      1 ¯1  0      ¯1  0  0
¯1  0  1     ¯1  0  1       0  0  0
 0  1  2      0  1 ¯1       0  0  1
```

16.22

```
    ∇Z←DT N
[1]  Z←N|T N∇

    ∇Z←T N
[1]  Z←¯1+⍳N
[2]  Z←Z∘.×Z∇

    ∇Z←CT N
[1]  Z←⌊(T N)÷N∇
```

a)

```
 DT 3      CT 3
0 0 0     0 0 0
0 1 2     0 0 0
0 2 1     0 0 1
```

b)

```
  DT 4         CT 4
0 0 0 0      0 0 0 0
0 1 2 3      0 0 0 0
0 2 0 2      0 0 1 1
0 3 2 1      0 0 1 2
```

c)

```
   DT 5           CT 5
0 0 0 0 0      0 0 0 0 0
0 1 2 3 4      0 0 0 0 0
0 2 4 1 3      0 0 0 1 1
0 3 1 4 2      0 0 1 1 2
0 4 3 2 1      0 0 1 2 3
```

d)

```
     DT 7               CT 7
0 0 0 0 0 0 0      0 0 0 0 0 0 0
0 1 2 3 4 5 6      0 0 0 0 0 0 0
0 2 4 6 1 3 5      0 0 0 0 1 1 1
0 3 6 2 5 1 4      0 0 0 1 1 2 2
0 4 1 5 2 6 3      0 0 1 1 2 2 3
0 5 3 1 6 4 2      0 0 1 2 2 3 4
0 6 5 4 3 2 1      0 0 1 2 3 4 5
```

e)

```
    PRODUCT        DIGIT        CARRY
   1  0 ¯1       1  0 ¯1       0 0 0
   0  0  0       0  0  0       0 0 0
  ¯1  0  1      ¯1  0  1       0 0 0
```

16.23 a) 5; ¯5; 10; ¯10; 0; 0
b) There is an error in the statement of this question: the word "binary" should by replaced by "decimal".

```
∇Z←NVDVAL X
Z←(¯1*X[1])×+/(1↓X)×10*⌽¯1+⍳⍴1↓X∇
```

716; ¯718

16.24 a) 24 10000; 134 100; 984 1000
b) .0024; 1.34; .984

16.25

```
  ∇Z←IFVD X
Z←(¯1+⍴IVDVAL X[2]),IVDVAL X[1]∇
```

16.26 a) 1 1 5 6; 2 2 2 2 7; 6 6 1 4 2 8 5 7
b) 5.6; 2.27; .142857

16.27 a) 510 90; 22500 9900; 1.42857E11 9.99999E11

17

17.1

EXPRESSION	SECTION	FUNCTION
`.00001<|X-Z*2`	12.1	SQT[3]
`X<Z*2`	12.2	Q[3]
`.00001<|X-Z*2`	12.2	Q[5]
`X<F Z`	12.2	GRF[2]
`.00001<|X-F Z`	12.2	GRF[3]
`X≠0`	12.3	GD[4]
`X[1]≠0`	12.3	GCD[3]
`X≥⍴Z`	12.4	BIN[2]

17.2 b) Substitute ∨ for ⌈ and ∧ for ⌊

17.4 1 1 0 1; 1 1 0 1; 0 0 1 0; 0 0 1 0; 0 1 1 0; 0 1 1 0

17.5 1 1 0 0; 1 0 1 0; 0 0 1 1; 0 0 0 1; 1 1 1 0; 0 0 0 1; 0 1 1 1; 1 0 0 0; 0 1 1 1; 1 1 0 1; 1 1 0 1; 1 0 0 1; 1 0 0 1

17.6

```
∇Z←P1 X
  Z←(0<X)∧(X<15)∧(0=2|X)∇

∇Z←P2 X
  Z←(X>0)∧0=X|24∇
```

P3 would be possible to define only for a certain desk at a certain time.

17.7 a) 0 0 0 0 0 0 0 0 0 0 0 0 1
0 1 0 1 0 1 0 1 0 1 0 1 0 0
0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0
0 0 0 1 1 1 1 0 1 0 1 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 1 0

b) 2 4 6 8 10 12 14;
1 2 4 6 8 12 24

17.8 a)

```
∇Z←P4 X
  Z←∨/X=2 3.5 7 8 13∇

∇Z←P5 X                    ∇Z←P6 X
 Z←∨/X='AEIOU'∇             Z←P5 X∇
```

0; 1; 0; 1; 0

17.9 a)

```
∇Z←P4V X
  Z←X∊2 3.5 7 8 13∇

∇Z←P5V X                   ∇Z←P6V X
 Z←X∊'AEIOU'∇               Z←P5V X∇
```

b) 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0; 2 3.5; 1 0 0 1 0 0 0 1 0 0 1 0 1 0; 1 0 0 1 0 0 0 1 0 0 1 0 1 0; *IIOOA*

17.10 a) 0 0 1 0 1; 0 0 1 1; 1 1 0 1 0; 1 1 0 0
b) 3 5; 5 3; 1 2 4; 9 7
c) There is an error in the statement; it should have referred to the expressions of part b) (the expressions of part a) do not produce sets). The expressions of part b) yield the following sets:
The set of all elements common to both *A* and *B*
The same set as above
The set of all elements of *A* which are not in *B*

The set of all elements of *B* which are not in *A*

17.11 a)

```
        S∘.×S
 4  6  8 10 12 14 16
 6  9 12 15 18 21 24
 8 12 16 20 24 28 32
10 15 20 25 30 35 40
12 18 24 30 36 42 48
14 21 28 35 42 49 56
16 24 32 40 48 56 64
```

0 0 1 0 1 0 1; 1 1 0 1 0 1 0;
2 3 5 7

b) 2 3 5 7 11 13 17 19
c) The function *PR* of Chapter 9 is equivalent to *F* since each generates the primes up to its argument.

17.12 a) 3 5 7 2; 7 2 3 5; 3 7 2 3 5 2; etc.
b) '*MEATS*'; '*TEAMS*'; '*MATES*'; '*TEAMMATES*'; etc.
c) All
d) All but the last
e) *I* must contain all the indices of *X*
f)

```
∇Z←X SAMESET I
 Z←(∧/(⍳⍴,X)∊I)∧(∧/I∊⍳⍴,X)∇
```

(the expression in the second pair of parentheses ensures that the expression *X*[*I*] does not yield a domain error.)

17.13 a) See the function *RD* in Exercise 17.14
b) 2 1 4 7 8; 3

17.15 a) 2 4 6; 6 4 2; 3 4 6; 3 4 6; 4 6; 4 6
b) Intersection is both associative and commutative.

17.16 a) 1 3 5; 10 8; 1 2 5; 12; 1 3 4 5 6; 1 5
b) Less is neither associative nor commutative.

17.17 a) 1 2 3 4 5 6 10 8; 10 8 6 4 2 1 3 5; 1 2 3 4 5 6 12; 3 4 6 12 1 2 5; 1 2 3 4 5 6 10 8 12; 1 2 3 4 5 6 10 8 12
b) Union is both associative and commutative.

17.18 a) *I* distributes over *U*
b) *U* distributes over *I*

17.19 c) 0 10 7 17 3 13 10 20 2 12 9 19 5 15 12 22; the first element of the foregoing (zero) is the sum over the empty vector selected by compression by the vector 0 0 0 0. This sum is defined to be zero because zero is the identity element of addition, that is, 0+*X* yields *X* for any *X*.
1 10 7 70 3 30 21 210 2 20 14 140 6 60 42 420; the first element (1) is the product over the empty vector and is the identity element of mutiplication.
d) The sum over each of the possible subsets of *T*; the product over each of the possible subsets of *T*

17.20 a)

```
        R                  T
    2 3 5                0 0 0
                         0 0 1
        N                0 1 0
   ¯2 ¯3 ¯5              0 1 1
                         1 0 0
        C                1 0 1
   ¯30 31 ¯10 1          1 1 0
                         1 1 1
```

b) 2 1 ¯1; ¯2; ¯2

18

18.1 3+4 5+.×V; ¯4+6 7+.×V; ¯4+7 6+.×V; ¯3+¯6 0+.×V; 3+¯6 0+.×V; ¯8+0 ¯9+.×V; ¯8+0 ¯9+.×V; ¯8+0 ¯9+.×V; 0+3 ¯6+.×V; 0+3 ¯6+.×V; 0+3 ¯6+.×V; 4+¯3 ¯7+.×V; 8+2 5 10+.×V; 8+2 0 10+.×V; ¯4+2 0 10+.×V; 18+0 0 10+.×V; 4+3 0 0+.×V; 4+3 0 0+.×V; 0+1 1 1+.×V; 0+4 2 1+.×V; 0+1 ¯1 1+.×V; 0+1 1 1 1+.×V

18.2 The results may not agree exactly with the original expression, but the evaluation of the original expression must agree with the evaluation of the original expression for any chosen set of argument values.

18.3 25; 28; 29; ¯15; ¯15; ¯26; ¯26; ¯26; ¯3; ¯3; ¯3; ¯19; 64; 54; 42; 58; 13; 13; 9; 20; 5; 24

18.4 a)

```
    A          B
  3 4        2 ¯4
            ¯3  2
```

b) ¯13 8; 9 ¯5; ¯9 10; 3 4; ¯13 20; 37 ¯19; ¯3 25

18.5 a) ¯3 6 and 2 2⍴4 ¯2 2 7
b) ¯5 45; 9 12; ¯9 27; ¯3 6; ¯23 12; 23 ¯37; ¯33 ¯33

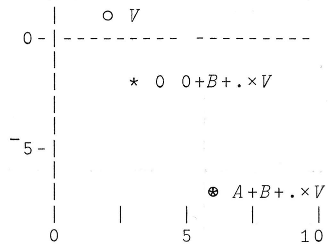
a) and b) Same as above

a) 0 0 and 2 2⍴3 7 8 4
b) 41 36; 9 24; 21 12; 0 0; 2 ¯24; ¯40 ¯4; ¯48 ¯84

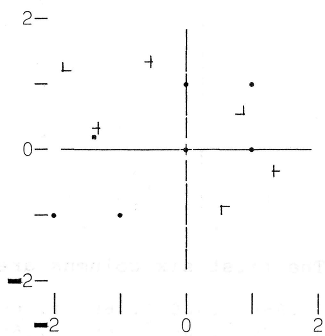
a) 2 8 and 2 2⍴3 0 0 7
b) 8 43; 11 8; 2 29; 2 8; ¯10 22; 11 ¯41; ¯25 ¯13

18.6 a) 18 ¯13 2 and 3 3⍴3 ¯4 7 0 2 0 0 3 ¯4

18.7 a) and 18.8 a)



18.9



18.10 The rotations (measured counter-clockwise in degrees) are:
270; 90; 180; 0; 315; 45

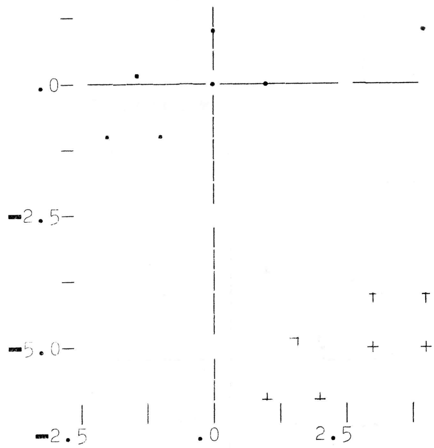
18.11 a) B+.×V produces a rotation of 300 degrees counter-clockwise (or 60 degrees clockwise). Each successive application of B produces a further clockwise rotation of 60 degrees
b) 6

18.12 b) The matrix is the same as one of the last two in Exercise 18.10

18.13 If R←B+.×⍉B then:

R[1;1] is (S×S)+(C×C) which is 1 (by definition of S and C)
R[1;2] is (S×-C)+(C×S) which is 0
R[2;1] is ((-C)×-C)+(C×S) which is 0
R[2;2] is ((-C)×-C)+(S×S) which is 1

18.14 a)



18.15 The first six columns are:

```
¯1.866 0 .866  .500  1.366 ¯1.366
 1.232 0 .500 ¯.866  ¯.366   .366
```

18.16 ⍉7 2⍴3 ¯5

18.17

```
         P+B+.×M
 2  3  4  3  4  2  3.2
¯3 ¯5 ¯5 ¯6 ¯6 ¯4 ¯3.6

         B+.×P+M
¯6 ¯5 ¯4 ¯5 ¯4 ¯6 ¯4.8
¯1 ¯3 ¯3 ¯4 ¯4 ¯2 ¯1.6

    (B+.×P)+(B+.×M)
¯6 ¯5 ¯4 ¯5 ¯4 ¯6 ¯4.8
¯1 ¯3 ¯3 ¯4 ¯4 ¯2 ¯1.6
```

18.18 b) If R←B+.×M then R[1;] equals B[1;1]×M[1;]and R[2;]equals B[2;2]×M[2;]

18.20 a) The answers given are only for the first element of each matrix:

```
(B11×C11)+(B12×C21)
(A11×(B11×C11)+(B12×C21))
    +(A12×(B21×C11)+(B22×C21))
(((A11×B11)+(A12×B21))×C11)
    +(((A11×B12)+(A12×B22))×C21)
```

b) Show that the second and third cases are equal by writing out all four terms of each and comparing. Follow same procedure for the other elements.

18.21 a) The answers given are only for the first element of each matrix:

```
(A11×(B11+C11))+A12×(B21+C21)
(A11×B11)+(A12×B21)+(A11×C11)
    +(A12×C21)
```

b) Show that the two case are equal by writing out all the terms and comparing.

18.22 b)

```
          B+.×M
1 2 3 ¯1 ¯2 0  1 1
0 0 0 ¯1 ¯2 0 ¯8 4
3 6 9  2  4 0 ¯2 1
```

18.24 a) Each point of the result is obtained by leaving the X coordinate unchanged and rotating the point in the Y-Z plane counter-clockwise by 45 degrees.

18.25 a)

```
          Y
 10 9 8 7 6 5 4 3 2 1 0

      M
  8    9   10   11   12   13   14   15   16   17   18
  6    7    8    9   10   11   12   13   14   15   16
  4    5    6    7    8    9   10   11   12   13   14
  2    3    4    5    6    7    8    9   10   11   12
  0    1    2    3    4    5    6    7    8    9   10
 ¯2   ¯1    0    1    2    3    4    5    6    7    8
 ¯4   ¯3   ¯2   ¯1    0    1    2    3    4    5    6
 ¯6   ¯5   ¯4   ¯3   ¯2   ¯1    0    1    2    3    4
 ¯8   ¯7   ¯6   ¯5   ¯4   ¯3   ¯2   ¯1    0    1    2
¯10   ¯9   ¯8   ¯7   ¯6   ¯5   ¯4   ¯3   ¯2   ¯1    0
¯12  ¯11  ¯10   ¯9   ¯8   ¯7   ¯6   ¯5   ¯4   ¯3   ¯2

      N
 10    9    8    7    6    5    4    3    2    1    0
  9    8    7    6    5    4    3    2    1    0   ¯1
  8    7    6    5    4    3    2    1    0   ¯1   ¯2
  7    6    5    4    3    2    1    0   ¯1   ¯2   ¯3
  6    5    4    3    2    1    0   ¯1   ¯2   ¯3   ¯4
  5    4    3    2    1    0   ¯1   ¯2   ¯3   ¯4   ¯5
  4    3    2    1    0   ¯1   ¯2   ¯3   ¯4   ¯5   ¯6
  3    2    1    0   ¯1   ¯2   ¯3   ¯4   ¯5   ¯6   ¯7
  2    1    0   ¯1   ¯2   ¯3   ¯4   ¯5   ¯6   ¯7   ¯8
  1    0   ¯1   ¯2   ¯3   ¯4   ¯5   ¯6   ¯7   ¯8   ¯9
  0   ¯1   ¯2   ¯3   ¯4   ¯5   ¯6   ¯7   ¯8   ¯9  ¯10

         0=M                  '□*'[1+0=M]      '□*'[1+0=N]
0 0 0 0 0 0 0 0 0 0 0         □□□□□□□□□□□      □□□□□□□□□□*
0 0 0 0 0 0 0 0 0 0 0         □□□□□□□□□□□      □□□□□□□□□*□
0 0 0 0 0 0 0 0 0 0 0         □□□□□□□□□□□      □□□□□□□□*□□
0 0 0 0 0 0 0 0 0 0 0         □□□□□□□□□□□      □□□□□□□*□□□
1 0 0 0 0 0 0 0 0 0 0         *□□□□□□□□□□      □□□□□□*□□□□
0 0 1 0 0 0 0 0 0 0 0         □□*□□□□□□□□      □□□□□*□□□□□
0 0 0 0 1 0 0 0 0 0 0         □□□□*□□□□□□      □□□□*□□□□□□
0 0 0 0 0 0 1 0 0 0 0         □□□□□□*□□□□      □□□*□□□□□□□
0 0 0 0 0 0 0 0 1 0 0         □□□□□□□□*□□      □□*□□□□□□□□
0 0 0 0 0 0 0 0 0 0 1         □□□□□□□□□□*      □*□□□□□□□□□
0 0 0 0 0 0 0 0 0 0 0         □□□□□□□□□□□      *□□□□□□□□□□

     (0=M)∨(0=N)                   (0=M)∧(0=N)
0 0 0 0 0 0 0 0 0 0 1          0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0          0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0          0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0          0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 1 0 0 0 0          0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 1 0 0 0 0 0          0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0          0 0 0 0 1 0 0 0 0 0 0
0 0 0 1 0 0 1 0 0 0 0          0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 1 0 0          0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 1          0 0 0 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0          0 0 0 0 0 0 0 0 0 0 0
```

b) The `1`'s in `0=M` each indicate a pair of values of *X* and *Y* for which the expression `(2×Y)+(X-12)` is equal to zero. Because these points lie on a straight line, they are said

to represent the line defined by the equation 0=(2×Y)+(X-12). Similar remarks apply to 0=N. The expression (0=M)∨(0=N) represents both lines and (0=M)∧(0=N) represents the point common to both lines, i.e., the solution to both of the equations 0=(2×Y)+(X-12) and 0=Y+¯1×X.

19

19.1 a) *Q*←*B*+.×*M*

Q

5	11	0	4	¯4	¯17	3	1
9	20	0	7	¯5	¯29	5	2

IB+.×*Q* is the same as *M*

P←*IB*+.×*M*

P

0	¯1	0	1	¯11	¯8	2	¯1
1	5	0	¯2	30	19	¯5	3

B+.×*P* is the same as *M*

b) *B*+.×*IB* and *IB*+.×*B* both yield the identity matrix

19.2 a) *B*+.×*M*

49	¯17	1	0	2	2	0
94	¯22	2	1	3	0	0
116	¯40	4	0	4	¯12	0

IB+.×*M*

1	¯.50	¯1	0	.50	10.50	0
2	9.75	¯1	1	¯.25	7.75	0
4	¯1.25	1	0	¯.25	¯9.25	0

b) *B*+.×*IB* and *IB*+.×*B* both yield the identity matrix.

19.3 a) 0; 0; 1; 0; 0; 0
b) 4.5 .5

19.4 *N*[;*J*]=*B*+.×*M*[;*I*] is true for the following values:
I: 1 2 3 4 5 6
J: 4 3 6 5 2 1

19.5 a) *V1*←5 ¯3 and *V2*←¯3 2
b) There is an error in the exercise: the expression for *N* should be *N*←(4×1 0)+(¯2×0 1). For the corrected exercise:
4 ¯2; 26 ¯16; 4 ¯2; 1
c) 4 ¯1; ¯39 25; 0 0; ¯35 21; ¯12 8

19.6 a) *V1*←¯1.5 3.5 and *V2*←1 ¯2
c) ¯.5 3.5; 12.5 ¯26.5; 0 0; 10.5 ¯24.5; 4 ¯8

19.7 a) 5 ¯3
b) 1

19.8 It is impossible to determine the basic solution with *VA*←0 0 because the first element of the result *B*+.×*VA* is also 0 and it cannot be divided into *VA* to give the basic solution.

19.9 *VA*, *K*, and *V1* are given for each matrix:

VA		*K*	*V1*	
3	¯7	¯2	¯1.50	3.50
8	¯2	10	.80	¯.20
8	6	100	.08	.06

19.10 a) *VA*←3 ¯2
K←¯1

b)

VA		*K*	*V2*	
2	¯4	2	1.00	¯2.00
3	¯2	¯10	¯.30	.20
6	¯8	¯100	¯.06	.08

19.11 a) *V1* and *V2* are given for each matrix:

V1		*V2*	
¯3.000	1.000	7.000	¯2.000
.550	¯.400	¯.150	.200
.050	.040	¯.025	.080
1.667	¯1.000	¯3.000	2.000

19.12 a) ¯1; 20; 200; 3
b) ¯2; 10; 100

19.13 a) *B*
8 5
4 3

b) 4

```
c) ¯8 ¯5        d) 100  0        e) 1 0
    4  3              0  1           0 1
```

19.14 a) Changes the sign
b) Changes the sign
c) Unchanged
e) Unchanged

19.15 a) 0
b) No
c) Make *M*[2;] equal to *M*[1;]

```
19.16  1.5 ¯3.5                  .75 ¯1
       ¯.5  1.5                ¯1.25  2
```

19.17 The matrices (in order across the page) contain *V1* and *V2* as columns:

```
¯3  7                           .55 ¯.15
 1 ¯2                          ¯.40  .20

.05 ¯.025                      1.67 ¯3
.04  .080                     ¯1.00  2
```

19.18 ¯41 18; ¯10.75 22.25; 82 ¯23; ¯.3 1.4; ¯.175 1.16; ¯34 23

19.19 21.25 ¯27.75; 4.5 ¯3.5; ¯9 16; ¯.75 1.25; 1.25 ¯1.75

```
19.20  a)   BS
            1 ¯2
           ¯2 4.5
```

```
b)         B+.×M
    13  47  9 ¯39  4  0 237
     6  22  4 ¯16  2  0 106
BS+.×B+.×M equals M
           BS+.×M
   ¯1.0 ¯7.0  1 ¯19 ¯2.0  0  19.0
    2.5 16.5 ¯2  41  4.5  0 ¯36.5
B+.×BS+.×M equals M
```

```
19.21  a) ¯.917  .583
            .667  .333

 .07  .03                      .12 ¯.04
¯.03  .13                     ¯.22  .24
```

19.22 All yield the identity matrix.

```
19.23   C←2 2⍴5 6 8 7
        D←2 2⍴3 5 4 2

  C+.×D                 D+.×C
39 37                  55 53
52 54                  36 38
```

```
19.24  a) 9 4 1 0
          4 2 0 1

1   .444   .111   0    r1 times ÷9
4  2.000   .000   1

1   .444   .111   0    add r1 times
0   .222  ¯.444   1      ¯4 to r2

1   .444   .111  .000  r2 times
0  1.000 ¯2.002 4.505    ÷.222

1  0   1.000 ¯2.000    add r2 times
0  1  ¯2.002  4.505     ¯.444 to r1

        BS
  1.000  ¯2.000
 ¯2.002   4.505
```

b) The results will be the same as the *BS* matrices formed in Exercise 19.21

19.25 a) Note: use same steps as in Ex. 19.24 a)

```
9  4    3
4  2  ¯11

1   .444     .333
4  2.000  ¯11.000

1   .444     .333
0   .222  ¯12.333

1   .444     .333
0  1.000  ¯55.506

1  0   25.000
0  1  ¯55.506

V← 25  ¯55.506
```

b) The results for *V* are: ¯9.167 5.667; ¯.120 ¯1.52; .8 ¯3.3

```
19.26  a) 4 4 0 1 0 0
          3 2 1 0 1 0
          2 1 0 0 0 1
```

```
1  1  0   .25  0  0     r1 times ÷4
3  2  1   .00  1  0
2  1  0   .00  0  1

1  1  0   .25  0  0
0 ¯1  1  ¯.75  1  0     r2+(r1×¯3)
0 ¯1  0  ¯.50  0  1     r3+(r1×¯2)

1  1  0   .25  0  0
0  1 ¯1   .75 ¯1  0     r2 times ÷¯1
0 ¯1  0  ¯.50  0  1

1  0  1  ¯.50  1  0     r1+(r2×¯1)
0  1 ¯1   .75 ¯1  0
0  0 ¯1   .25 ¯1  1     r3+(r2×1)

1  0  1  ¯.50  1  0
0  1 ¯1   .75 ¯1  0
0  0  1  ¯.25  1 ¯1     r3 times ÷¯1

1  0  0  ¯.25  0  1     r1+(r3×¯1)
0  1  0   .50  0 ¯1     r2+(r3×1)
0  0  1  ¯.25  1 ¯1
      BS
¯.25  0  1
 .50  0 ¯1
¯.25  1 ¯1
```

b) The 3-by-3 identity matrix
c) 5.5 ¯5 ¯11.5

19.27 a) Only the solution matrices are given:

```
  .1284   .2202  ¯.1193
 ¯.1743  ¯.1560   .3761
  .2844   .2018  ¯.1927

  .0735   .0475  ¯.0243
 ¯.0133   .0544  ¯.0035
  .0029  ¯.0336   .0660
```

c) ¯1.560 2.689 ¯1.597; ¯.236 ¯.319 .569

```
19.28   10   3  14  12
         2  12   1   3
        ¯4   7  15  14

 1     .3    1.4    1.2     r1 times ÷10
 2   12.0    1.0    3.0
¯4    7.0   15.0   14.0

1      .3    1.4    1.2
0    11.4   ¯1.8     .6     r2+(r1×¯2)
0     8.2   20.6   18.8     r3+(r1×¯4)

1    .3    1.400    1.200
0   1.0    ¯.158     .053   r2×(÷11.4)
0   8.2   20.600   18.800

1  0    1.447    1.184   r2+(r2×¯.3)
0  1    ¯.158     .053
0  0   21.895   18.368   r3+(r2×¯8.2)

1  0   1.447   1.184
0  1   ¯.158    .053
0  0   1.000    .839   r3×(÷21.895)

1  0  0   ¯.030   r1+(r3×¯1.447)
0  1  0    .185   r2+(r3×.158)
0  0  1    .839

V← ¯.030 .185 .839
```

```
19.29    .069    .021   ¯.066
        ¯.014    .083    .007
         .025   ¯.033    .046
```

19.30

```
      ∇Z←F X
  [1] NM←2 2⍴1 ¯1 ¯1 1
  [2] Z←NM×⊖⌽⍉X
  [3] Z←Z÷DET X∇
```

19.31

```
    ∇Z←G X
[1] N←3
[2] Z←X,(⍳N)∘.=⍳N
[3] C←1
[4] Z[C;]←Z[C;]×÷Z[C;C]
[5] D←(C≠⍳N)/⍳N
[6] Z[D;]←Z[D;]-Z[D;C]∘.×Z[C;]
[7] C←C+1
[8] → 4 9[1+C>N]
[9] Z←Z[⍳N;N+⍳N]∇
```

19.32 To modify G so its argument can be a square matrix of any dimension, change line [1]to `N←1↑⍴X`

19.34 `M←X∘.*¯1+⍳⍴X`

```
      M                 ⌹M
1  1   1        1.714   ¯.800    .086
1  3   9        ¯.786    .900   ¯.114
1  8  64         .071   ¯.100    .029

    C←(⌹M)+.×Y           C←Y⌹M
    C                    C
0 .5 .5                0 .5 .5
```

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